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FOR

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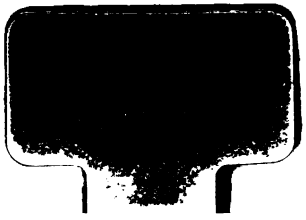

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BY

JAMES HADDON, M.A.

ARITHMETICAL MASTER
AND SECOND MATHEMATICAL MASTER OF KING'S COLLEGE SCHOOL,
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PREFACE.

THIS little work is divided into three Sections, with a view to prevent loss of time to the student. After making himself master of the first section, he may enter upon the subject of the second or third, according as it may be desirable for him to direct his attention to a Mathematical or to a Commercial course of study.

The solutions of many examples are given at length, and to these the pupil is recommended to pay particular attention. Of the other examples, a certain number are given with the answers, and the rest without them, in the hope that the student will thus be induced to rely in some measure upon himself for the accuracy of his work.

In order to facilitate and encourage frequent self-examination, numerous questions have been introduced at intervals throughout the work, which it is hoped the student will endeavour to answer before he proceeds to the consideration of the succeeding subjects. It will be advantageous to him, from time to time, to make use of the Examination Papers at the end of the book; some of which he will find, at every stage, practically useful, as tests of the soundness of his progress, and of his comprehension of the subject.

CONTENTS.

SECTION I.

SECTION II.

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ARITHMETIC.

SECTION I.

ARITHMETIC is the science of computing by numbers.

Number is either an unit, or a collection of units; as one man, ten men.

An integer, or whole number, is any whole quantity or number; as a pound, three oranges, seven yards, &c.

A prime number is that which cannot be divided by any number excepting itself or an unit, without a remainder, as three, seven, eleven, &c.

A composite number is that which may be produced by multiplying two or more numbers together; as four, fifteen, twenty-two, &c.

The numbers which are multiplied together to produce a composite number are called its factors, or component parts.

NOTATION.

By Notation, or Numeration, we learn to express numbers by words or figures, or to read or write any number.

The characters used for this purpose, are—1, one; 2, two; 3, three; 4, four; 5, five; 6, six; 7, seven; 8, eight; 9, nine; 0, cipher.

These figures have various values, depending upon their situation, as in the following Table, which must be read first from right to left, and then from left to right.

&c.	Hundreds of millions.	Tens of millions.	Millions.	Hundreds of thousands.	Tens of thousands.	Thousands.	Hundreds.	Tens.	Units.
	9	8	7	6	5	4	3	2	1
		9	8	7	6	5	4	3	2
			9	8	7	6	5	4	3
				9	8	7	6	5	4
					9	8	7	6	5
						9	8	7	6
							9	8	7
								9	8
									9

The uppermost line is nine hundred and eighty-seven millions, six hundred and fifty-four thousand, three hundred and twenty-one: the fifth line is ninety-eight thousand seven hundred and sixty-five.

603 is six hundred and three.

40025 is forty thousand and twenty-five.

Examples.

1. Write the following numbers in words:—

37, 370, 2004, 20004, 555, 5555, 260485, 26048500.

2. Write the following numbers in figures:—

Seventy-three. Seven hundred and thirty. Seven hundred and three. Four thousand and ten. Forty thousand one hundred.

3. How many figures must be used to express

Tens of thousands?
Millions?
Hundreds?
Hundreds of millions?

ADDITION.

By Addition we find the *sum* of several numbers.

The sign of addition is + plus (more).

The sum of 3, 4 and 12, is 19, and may be expressed thus, $3 + 4 + 12 = 19$; that is, 3 plus 4 plus 12 equals 19.

RULE.

Place the numbers under each other; thus, units under units, tens under tens, &c. Add up the figures in the column of units; consider how many tens are contained in their sum; set down the units that remain, and carry the tens to the next column: proceed thus till the whole is finished.

Example. Add together the following numbers:—

3426
91048
7004
85768
465
894
26998

215603 = the sum, or answer.

The sum of the first column is 43 units, which is 4 tens and 3 units; the 3 units are set down, and the 4 tens carried to the second column, whose sum is 40 tens, which is 4 hundreds, and as there are no tens over, a cipher is set down, and the 4 hundreds carried to the third column, whose sum is 36 hundreds, which is 3 thousands and 6 hundreds; the 6 hundreds are set down, and the 3 thousands carried to the fourth column, and so on.

Method of Proof.

Cut off the top number, add the other numbers downwards; and set their sum under the answer, then add this sum to the top number, and if the result agree with the answer, the work is correct.

Examples.

1. Add together 4, 18, 36, 5, 27 and 42, and prove your work.

2. Add together 243, 608, 143, 2410 and 583, and prove your work.

3. Add together 7243, 14680, 374, 4236 and 37, and prove your work.

4. Add together four thousand two hundred and twenty-six, fifty-one thousand and seventy-three, four hundred and twenty-thousand and sixty-five, and nine hundred and ten, and prove your work.

5. There are four purses, the first containing 423 sovereigns, the second 1207, the third 69, and the fourth seven thousand and nineteen: how many sovereigns are there altogether?

6. A grazier buys of one person forty-nine sheep, of another three hundred and sixty, of another fifty, of another two hundred and two, of another one hundred and eleven, and of another a thousand and forty-three: how many sheep did he buy?

7. What is Arithmetic? What is an integer? What is a composite number? What is numeration? How many figures are necessary to express tens of thousands? What is the object of addition? Write the sign of addition. What is the sum of 6, 8 and 3? What is 15 + 10 equal to?

SUBTRACTION.

By Subtraction we find the *difference* between two numbers, by taking the less from the greater.

The sign of subtraction is — minus (less).

The difference between 12 and 8 is 4, and may be expressed thus, 12 — 8 = 4; that is, 12 minus 8, equals 4.

RULE.

Place the less number under the greater, thus, units under units, tens under tens, &c.; begin with the units; take each figure in the lower line from the figure above it, and set down the remainder. If the lower figure be greater than the upper one, subtract the lower from 10, and add the remainder to the upper figure; carry one to the next lower figure, and proceed thus to the end.

Example. From 7613058 subtract 1087593.

7613058
1087593

6525465 = the difference.

3 units taken from 8 units leave 5 units, the 5 units are set down: 9 tens taken from 10 tens leave 1 ten, which, added to the 5 tens above, makes 6 tens; the 6 tens are set down, and 1 hundred carried to the next lower figure, thus, 1 hundred and 5 hundreds are 6 hundreds; 6 hundreds taken from 10 hundreds leave 4 hundreds; the 4 hundreds are set down, and 1 thousand carried to the next lower figure; and so on.

In the column of tens we take 9 tens from 10 tens, because 9 tens cannot be taken from 5 tens; and since the 10 tens are afterwards added, in the shape of 1 hundred, to the next lower figure, we do, in effect, by this operation, add 1 hundred to both the given numbers. Similarly, in the column of hundreds, we take 6 hundreds from 10 hundreds, and add the 10 hundreds, in the shape of 1 thousand, to the next lower figure, the 7 thousands; this operation, therefore, is in reality adding 1 thousand to both the given numbers.

In the above example, 1 hundred, 1 thousand, 1 ten thousand, and 1 hundred thousand are successively added to each of the given numbers; and the accuracy of the result depends upon the fact, that the difference of two numbers remains the same when each of them is increased by the addition of any given number.

Method of Proof.

Add the two lower numbers together, and if their sum be equal to the uppermost number, the work is correct.

Examples.

1. From 4801637 take 2400315, and prove your work.
2. From 5740189 subtract 187569, and prove your work.
3. Find the difference between 7308651 and 628753.
4. How much is 527108 greater than 20489?
5. From forty-nine thousand and seven, subtract thirty thousand nine hundred and fifty, and prove the work.
6. A person was born in the year 1769, and died in 1841: how many years did he live?
7. A has five houses, and B four. A's houses are severally worth 540*l.*, 685*l.*, 372*l.*, 945*l.*, and 1032*l.* B's are severally worth 407*l.*, 863*l.*, 2000*l.*, and 1500*l.*: which is the richer man, A or B, and by how much? Prove each operation.

8. What is number? What is the difference between prime numbers and composite numbers? What do we find by subtraction? What is the difference between 17 and 9?

What is the sum of 11 and 3? What is 18 — 12 equal to?

What is 3 + 9 equal to?

MULTIPLICATION.

By Multiplication we find what a number amounts to if repeated a certain number of times; it is a short method of addition.

The number produced by multiplication is called the *product.*

The number multiplied is the multiplicand.

The number multiplied by is the multiplier.

The multiplicand and multiplier are called factors.

The sign of multiplication is ×.

The product of 11 and 5 is 55, and may be expressed thus, 11 × 5 = 55; that is, 11 multiplied by 5 equals 55: 11 being the multiplicand, 5 the multiplier and 55 the product.

MULTIPLICATION TABLE.

Twice	Three	Four	Five	Six	Seven
1 are 2	1 are 3	1 are 4	1 are 5	1 are 6	1 are 7
2 ... 4	2 ... 6	2 ... 8	2 ... 10	2 ... 12	2 ... 14
3 ... 6	3 ... 9	3 ... 12	3 ... 15	3 ... 18	3 ... 21
4 ... 8	4 ... 12	4 ... 16	4 ... 20	4 ... 24	4 ... 28
5 ... 10	5 ... 15	5 ... 20	5 ... 25	5 ... 30	5 ... 35
6 ... 12	6 ... 18	6 ... 24	6 ... 30	6 ... 36	6 ... 42
7 ... 14	7 ... 21	7 ... 28	7 ... 35	7 ... 42	7 ... 49
8 ... 16	8 ... 24	8 ... 32	8 ... 40	8 ... 48	8 ... 56
9 ... 18	9 ... 27	9 ... 36	9 ... 45	9 ... 54	9 ... 63
10 ... 20	10 ... 30	10 ... 40	10 ... 50	10 ... 60	10 ... 70
11 ... 22	11 ... 33	11 ... 44	11 ... 55	11 ... 66	11 ... 77
12 ... 24	12 ... 36	12 ... 48	12 ... 60	12 ... 72	12 ... 84

MULTIPLICATION TABLE—*continued.*

Eight	Nine	Ten	Eleven	Twelve
1 are 8	1 are 9	1 are 10	1 are 11	1 are 12
2 ... 16	2 ... 18	2 ... 20	2 ... 22	2 ... 24
3 ... 24	3 ... 27	3 ... 30	3 ... 33	3 ... 36
4 ... 32	4 ... 36	4 ... 40	4 ... 44	4 ... 48
5 ... 40	5 ... 45	5 ... 50	5 ... 55	5 ... 60
6 ... 48	6 ... 54	6 ... 60	6 ... 66	6 ... 72
7 ... 56	7 ... 63	7 ... 70	7 ... 77	7 ... 84
8 ... 64	8 ... 72	8 ... 80	8 ... 88	8 ... 96
9 ... 72	9 ... 81	9 ... 90	9 ... 99	9 ... 108
10 ... 80	10 ... 90	10 ... 100	10 ... 110	10 ... 120
11 ... 88	11 ... 99	11 ... 110	11 ... 121	11 ... 132
12 ... 96	12 ... 108	12 ... 120	12 ... 132	12 ... 144

The table is to be learnt thus,

Twice 1 are 2, twice 2 are 4, &c.; 3 times 1 are 3, 3 times 2 are 6, &c.

RULE.

Place the multiplier under the multiplicand; thus, units under units, tens under tens, &c.

Multiply each figure of the multiplicand, beginning with the units, by the right-hand figure of the multiplier; set the first figure of the product under the figure multiplied by, and carry as in addition.

Proceed in a similar manner with the other figures of the multiplier.

Add the several products together.

Example.—Multiply 423 by 142.

423 multiplicand
142 multiplier

846
1692
423

60066 product, or answer.

3 units multiplied by 2 units are equal to 6 units, the 6 units are set down in the units place; 2 tens multiplied by 2 units are equal to 4 tens; the 4 tens are set down in the tens place; 4 hundreds multiplied by 2 units are equal to 8 hundreds, the 8 hundreds are set down in the hundreds place; the first product is 846 units.

3 units × 4 tens = 12 tens or 1 hundred and 2 tens, the 2 tens are set down in the tens place and the 1 hundred carried on; 2 tens × 4 tens = 8 hundreds, which together with the 1 hundred carried make 9 hundreds; the 9 is set down in the hundreds place; 4 hundreds × 4 tens = 16 thousands or 1 ten thousand and 6 thousands; the 6 is set down in the thousands place and the 1 in the tens of thousands place: the second product is 1692 tens.

3 units × 1 hundred = 3 hundreds, the 3 hundreds are set down in the hundreds place; 2 tens × 1 hundred = 2 thousands, the 2 thousands are set down in the thousands place; 4 hundreds × 1 hundred = 4 tens of thousands, the 4 is set down in the tens of thousands place: the third product is 423 hundreds.

The three products being added together, their sum is 60066, which is the product of 423 and 142.

When there are ciphers at the right-hand of either or both of the given numbers, write them down on the right-hand of the answer.

When there are ciphers in any other part of the multiplier, neglect them.

To multiply by 10, 100, 1000, &c., attach as many ciphers to the right of the multiplicand as there are in the multiplier; thus 257 × 100 = 25700.

Method of Proof.

Make the multiplicand the multiplier, and the multiplier the multiplicand, and multiply again.

When the multiplier is a composite number, neither of whose factors exceeds 12, multiply first by one of these factors, and then the product thus obtained by the other. As a method of proof in this case make the factors or multipliers change places, and multiply again.

Example.—Multiply 4083 by 35.

4083	4083
5	7
20415	28581
7	5
142905 product, or answer.	142905 proof.

Examples.

1. Multiply 604328 by 2. Work this twice.
2. Multiply 485736 by 3, and prove your work by addition.
3. Multiply 250608 by 4. Find a way to prove this without adding.
4. Multiply 865837 by 5; also by 6, 7, 8, 9, 10, 11 and 12, and prove each operation.
5. Multiply 5068 by 365, and prove your work.
6. Multiply 395076 by 15, also by 18, 21, 25 and 36 by component parts, and prove each operation.
7. Multiply 7082490 by 203, also by 500, 4080, 7200 and 59007, performing each operation twice.
1. A man earns 136 pounds in 1 year; how many pounds does he earn in 20 years? Do this in two ways.
9. The crew of a frigate consists of 847 men, and each man receives 3*l.* a month; how much is paid to the whole crew in 12 months? Do this in two ways.
10. If a furnace consumes 12 tons of coal in a week, how many tons does it consume in 3 years, each year being 52 weeks? Do this in two ways.
11. What is the object of multiplication? What do you call the number produced by multiplication? Write the signs of addition, subtraction, and multiplication. State which of the following numbers are prime, and which are composite numbers; and of the composite ones name the component parts, 7, 16, 22, 23, 54, 61, 73, 108.

DIVISION.

By Division we find how many times one number is contained in another; it is a short method of subtraction.

The number resulting from division is called the *quotient*.

The number divided is the dividend.

The number we divide by is the divisor.

The sign of division is $\div$

The quotient resulting from the division of 20 by 5 is 4, and may be thus expressed, $20 \div 5 = 4$, or $\frac{20}{5} = 4$, that is, 20 divided by 5 equals 4; 20 being the dividend, 5 the divisor, and 4 the quotient.

RULE. *When the divisor does not exceed* 12,

Place the divisor on the left of the dividend, with a curve between them.

Divide each figure of the dividend, beginning at the left, by the divisor, and set down the quotient.

If, when dividing any figure, there be a remainder, carry it as so many tens to the next figure of the dividend, continue the division, and set down the quotient as before.

Example. Divide 6749 by 5.

Divisor 5) 6749 dividend.

1349$\frac{4}{5}$ quotient.

Proof by multiplication.

1349 4

5

6749

The operation is practically performed thus: fives in 6 once and 1 over, set down 1 and carry 1; fives in 17 three times and 2 over, set down 3 and carry 2; fives in 24 four times and 4 over, set down 4 and carry 4; fives in 49 nine times and 4 over, set down 9. The remainder 4 placed over the divisor forms the fraction four-fifths.

The 5 is contained in 6749, 1 thousand times, and there is 1749 over.

The 5 is contained in 1749, 3 hundred times, and there is 249 over.

The 5 is contained in 249, 40 times, and there is 49 over,

The 5 is contained in 49, 9 times, and there is 4 over.

Hence, the 5 is contained altogether in 6749, 1 thousand 3 hundred forty and nine and four-fifths times.

When the divisor has ciphers on the right hand, and the significant figures on the left hand do not exceed 12, cut off the ciphers, and the same number of figures from the right of the dividend.

To divide by 10, 100, 1000, &c., cut off as many figures from the right of the dividend as there are ciphers in the divisor. The figures cut off form the remainder, the others the quotient.

Example. Divide 572068 by 900.

9,00) 5720,68

quotient $635 - \frac{568}{900}$

Proof by multiplication.

635 - 568
900
571500
568
572068

When the divisor is a composite number,

Divide first by one of its factors, and then the result thus obtained by the other.

To find the true remainder.

Multiply the last remainder by the preceding divisor, and add the preceding remainder.

Example. Divide 60457 by 45.

5) 60457

9) 12091 - 2

quotient 1343 - 4 } 22 true remainder.

9

12091

5 } Proof by multiplication.

60457

When the divisor exceeds 12,

See how often the first figure of the divisor is contained in the first figure or in the first two figures of the dividend, and set the quotient on the right hand of the dividend.

Multiply the divisor by this quotient figure.

Subtract the product from the dividend, and bring down the next figure of the dividend to the remainder.

See how often the divisor is contained in this remainder, and proceed as before.

Example. Divide 15732 by 13.

13) 15732 ($1210\frac{2}{13}$ quotient.

13	Proof by multiplication.
—	1210 - 2
27	13
26	—
—	3632
13	1210
13	—
—	15732

2 remainder.

Examples. To be proved by multiplication.

1. Divide 184627 by 2, also by 3, 4, 5, and 6.
2. Divide 703806 by 7, also by 8, 9, 10, 11, and 12.
3. Divide 580731 by 16, also by 20, 27, 30, 81, 90, and 121.
4. Divide 174083 by 13, also by 23,740, 8300, and 543.
5. Divide 721460 by 42, also by 498, 700, 1804, and 72.
6. Divide 252 by 63, and prove it by subtraction.
7. 1200 workmen receive altogether 842,400*l.*; if this sum be equally divided between them, how much will each man receive?
8. A gentleman leaves by will thirty thousand pounds, thus; to his widow one-third, and the remainder equally between his four children: how much will each receive?
9. What is the object of division? What is a dividend, a quotient, a divisor? How is division proved? How is subtraction proved? Write several examples similar to the following; $7-3=4$, $12\times 8=96$, $11+7=18$, $63\div 7=9$. What do we find by subtraction, by addition, by multiplication? What effect is produced upon a number by multiplying it by 2 and then dividing the product by 10? What effect is produced upon the quotient by previously dividing both divisor and dividend by the same number?

TABLE OF ENGLISH MONEY.

4 farthings	=	1 penny
12 pence	=	1 shilling
20 shillings	=	1 pound
21 shillings	=	1 guinea
5 shillings	=	1 crown.

Pence Table.				Shillings Table.			
d.		*s.*	*d.*	*s.*		£	*s.*
20	are	1	8	20	are	1	0
24	...	2	0	30	...	1	10
30	...	2	6	40	...	2	0
36	...	3	0	50	...	2	10
40	...	3	4	60	...	3	0
48	...	4	0	70	...	3	10
50	...	4	2	80	...	4	0
60	...	5	0	90	...	4	10
70	...	5	10	100	...	5	0
72	...	6	0	110	...	5	10
80	...	6	8	120	...	6	0
84	...	7	0	130	...	6	10
90	...	7	6	140	...	7	0
96	...	8	0	150	...	7	10
100	...	8	4	160	...	8	0
108	...	9	0	170	...	8	10
120	...	10	0	180	...	9	0

TROY, OR GOLDSMITH'S WEIGHT.

4 grains (gr.) = 1 carat
24 grains = 1 pennyweight (dwt.)
20 dwts. = 1 ounce (oz.)
12 ounces = 1 pound (lb.)

By this weight gold, silver, and precious stones are weighed.

AVOIRDUPOIS WEIGHT.

16 drams (dr.) make 1 ounce (oz.)
16 ounces ... 1 pound (lb.)
14 pounds ... 1 stone (st.)
28 pounds, or 2 stones ... 1 quarter (qr.)
4 quarters, or 8 st., or 112 lbs. ... 1 hundred (cwt.)
20 cwt. ... 1 ton.

By this weight all coarse and heavy goods are weighed, and all metals, except gold and silver.

APOTHECARIES WEIGHT.

20 grains (gr.) make 1 scruple ℈
3 scruples ... 1 dram ʒ
8 drams ... 1 ounce ℥
12 ounces ... 1 pound ℔.

By this weight apothecaries compound their medicines, but they buy drugs by avoirdupois weight.

LONG MEASURE.

3 barleycorns	make	1 inch
12 inches	...	1 foot
3 feet	...	1 yard
6 feet	...	1 fathom
5½ yards	...	1 pole, or perch
40 poles, or 220 yds.	...	1 furlong
8 furlongs, or 1760 yds.	...	1 mile
3 miles	...	1 league
60 geographical, or 69½ statute miles	...	1 degree
4 inches	...	1 hand
7·92 inches	...	1 link
100 links	...	1 chain.

SQUARE, OR SUPERFICIAL MEASURE.

144 inches	make	1 foot
9 square feet	...	1 square yard
30¼ square yards	...	1 square pole, or perch
40 square poles	...	1 rood
4 roods, or 4840 sq. yards, or 10 sq. chains	...	1 acre
640 acres	...	1 square mile.

CUBIC, OR SOLID MEASURE.

1728 cubic inches	make	1 cubic foot
27 cubic feet	...	1 cubic yard
40 cubic feet of rough, or 50 cubic feet of hewn timber	...	1 load
277·274 cubic inches	...	1 imperial gallon.

LIQUID MEASURE.

4 gills	make	1 pint
2 pints	...	1 quart
4 quarts	...	1 gallon
9 gallons	...	1 firkin of beer
10 gallons	...	1 anker of spirits
18 gallons	...	1 kilderkin
36 gallons	...	1 barrel of beer
42 gallons	...	1 tierce of wine
54 gallons	...	1 hogshead of beer
63 gallons	...	1 hogshead of wine
2 hogsheads	...	1 pipe of wine
2 pipes	...	1 tun.

DRY MEASURE.

4 gills	make	1 pint
2 pints	...	1 quart
2 quarts	...	1 pottle
4 quarts	...	1 gallon
2 gallons	...	1 peck
4 pecks	...	1 bushel
4 bushels	...	1 coomb
8 bushels	...	1 quarter
5 quarters	...	1 wey
2 weys	...	1 last.

CLOTH MEASURE.

2¼ inches	make	1 nail
4 nails	...	1 quarter of a yard
3 quarters	...	1 Flemish ell
4 quarters	...	1 yard
5 quarters	...	1 English ell
6 quarters	...	1 French ell.

TIME.

60 seconds	make	1 minute
60 minutes	...	1 hour
24 hours	...	1 day
7 days	...	1 week
4 weeks	...	1 month
12 calendar months, or 13 common months 1 day 6 hours, or 365 days 6 hours	...	1 year.

ANGULAR MEASURE.

63 seconds (″)	=	1 minute (′)
60′	=	1 degree (°)
30°	=	1 sign of the Zodiac
90°	=	1 quadrant
12 signs, or 360°	=	1 great circle.

COAL MEASURE.

4 pecks	make	1 bushel
8 bushels	...	1 sack
3 sacks	...	1 vat, or strike
12 sacks	...	1 chaldron
21 chaldrons	...	1 score.

WOOL WEIGHT.

7 pounds	make	1 clove
2 cloves	...	1 stone
2 stones	...	1 tod
6½ tods	...	1 wey
2 weys	...	1 sack
12 sacks	...	1 last
240 pounds	...	1 pack.

HAY AND STRAW WEIGHT.

36 pounds	make	1 truss of straw
56 pounds	...	1 truss of old hay
60 pounds	...	1 truss of new hay
36 trusses	...	1 load.

The standard for gold coin in England is 22 parts of pure gold and 2 parts of copper; for silver coin, 37 parts of pure silver and 3 parts of copper.

One pound troy of gold is coined into $46\frac{2}{40}$ sovereigns.

One pound troy of silver is coined into 66 shillings.

One pound avoirdupois of copper is coined into 24 pence.

The Mint price of standard gold is 3*l.* 17*s.* 10½*d.* per ounce, and of silver 5*s.* 6*d.* per ounce.

177 lbs. troy = 144 lbs. avoirdupois
1 lb. troy = 1 lb. apothecaries.

Land is measured by a chain 4 poles or 22 yards long.

A piece of ground 10 chains long and 1 chain broad = 1 acre.

Painting, glazing, roofing, plastering, &c., are measured by square measure.

A square inch is a square 1 inch long and 1 inch broad.

A square foot is 12 inches long and 12 inches broad.

A cubic inch is a solid body having six equal sides, each side being one square inch.

Stone, timber, and all solids are measured by cubic measure.

One cubic inch of water weighs 252·458 grains.

Any year of our Lord, within the century, divisible by 4 without remainder, is leap year. If there be a remainder, it shows the number of years after leap year; thus the year 1849 divided by 4 gives remainder 1, hence it is the year after leap year.

12 articles	make	1 dozen
20 articles	...	1 score
12 dozen	...	1 gross
24 sheets of paper	...	1 quire
20 quires	...	1 ream.

COMPOUND ADDITION.

By compound addition we find the *sum* of several numbers consisting of different names.

RULE.

Place the numbers so that those of the same name stand under each other.

Add up the numbers in the lowest name, and find, by division, how many units of the next higher name are contained in their sum.

Set down the remainder, and carry the quotient to the next higher name. Proceed thus to the end.

The method of proof is the same as in addition of simple numbers.

Example. Add together 41*l.* 14*s.* 6¼*d.*, 280*l.* 12*s.* 4¾*d.*, 648*l.* 0*s.* 10½*d.*, 67*l.* 18*s.* 5*d.*, 1*l.* 13*s.* 0½*d.*, and 18*s.* 11¾*d.*

£	*s.*	*d.*	
41	14	6¼	
280	12	4¾	
648	0	10½	
67	18	5	
1	13	0½	
0	18	11¾	
1040	18	2½	= sum or answer.
999	3	8¼	
1040	18	2½	proof.

The sum of the column of farthings is 10 farthings, which is 2 pence and 2 farthings; the two farthings are set down, and the 2 pence carried to the column of pence, whose sum is 38 pence, which is 3 shillings and 2 pence; the 2 pence are set down, and the 3 shillings carried to the column of shil

lings, whose sum is 78 shillings, which is 3 pounds and 18 shillings; the 18 shillings are set down, and the 3 pounds carried to the column of pounds, whose sum is 1040 pounds.

Examples.

(1.) £	s.	d.
41	6	$4\frac{1}{3}$
6	7	$3\frac{1}{4}$
19	8	$5\frac{3}{4}$
30	0	6
10	3	$7\frac{1}{3}$
4	9	$2\frac{1}{4}$

(2.) £	s.	d.
25	9	$4\frac{3}{4}$
50	0	$7\frac{1}{2}$
66	8	$6\frac{1}{4}$
80	4	$3\frac{1}{2}$
17	19	$11\frac{3}{4}$
46	17	$0\frac{1}{4}$

(3.) £	s.	d.
300	9	6
150	18	$7\frac{1}{2}$
75	9	$4\frac{3}{4}$
160	7	$11\frac{1}{4}$
80	8	$9\frac{1}{2}$
97	14	$10\frac{3}{4}$

(4.) £	s.	d.
560	7	9
320	9	$4\frac{1}{2}$
697	14	$6\frac{3}{4}$
400	16	$3\frac{1}{4}$
99	19	11
176	18	$1\frac{3}{4}$

(5.) £	s.	d.
576	16	11
896	19	$10\frac{3}{4}$
247	14	$0\frac{1}{2}$
970	0	$7\frac{1}{4}$
262	12	$9\frac{3}{4}$
476	17	$8\frac{1}{2}$

(6.) £	s.	d.
109	5	$6\frac{1}{4}$
769	14	$5\frac{3}{4}$
916	9	4
433	10	8
231	9	$5\frac{1}{2}$
528	10	$6\frac{1}{2}$

(7.) £	s.	d.
1076	4	5
976	16	$7\frac{3}{4}$
896	17	$11\frac{1}{2}$
1967	4	$5\frac{1}{4}$
871	2	$4\frac{3}{4}$
1561	19	$4\frac{1}{4}$

(8.) £	s.	d.
1027	5	9
4568	11	$11\frac{1}{2}$
5624	15	$9\frac{3}{4}$
562	19	$10\frac{1}{4}$
1056	18	$7\frac{3}{4}$
9067	17	$6\frac{1}{2}$

(9.) £	s.	d.
4276	9	$2\frac{3}{4}$
1378	14	$9\frac{1}{2}$
5689	19	7
126	4	$3\frac{1}{4}$
7896	17	2
8468	15	$5\frac{3}{4}$

(10.)

£	s.	d.
1276	19	11¾
4799	17	10½
9786	15	4¾
4656	13	2¼
2568	16	3½
3469	2	9½

(11.)

£	s.	d.
9156	0	11½
4320	5	0¼
7698	17	6¾
4896	3	9¼
5678	19	8½
9012	15	7¾

(12.)

£	s.	d.
5678	7	1½
19675	19	4¾
4178	15	7½
8956	18	10¼
6857	19	11½
9999	9	9¾

TROY WEIGHT.

(13.)

lb.	oz.	dwt.	gr.
17	5	16	23
15	6	19	21
13	7	15	19
26	11	13	9
14	2	0	11
20	11	19	23

(14.)

lb.	oz.	dwt.	gr.
27	11	7	2
15	4	19	21
9	3	17	15
11	4	16	12
13	2	11	13
15	11	2	17

AVOIRDUPOIS WEIGHT.

(15.)

cwt.	qr.	lb.
1	3	27
14	2	20
7	1	12
9	2	19
7	1	21
0	3	26

(16.)

lb.	oz.	drs.
14	13	14
9	15	12
7	14	11
11	9	9
12	12	7
6	9	8

APOTHECARIES WEIGHT.

(17.) lb.	oz.	drs.	scr.
2	11	7	2
14	10	7	1
9	7	6	2
15	9	7	1
19	4	6	2
17	11	7	1

(18.) oz.	dr.	scr.	gr.
9	7	2	19
7	1	0	17
11	5	1	16
6	3	2	5
4	6	1	14
3	1	2	9

LONG MEASURE.

(19.) miles.	furlongs.	poles.
2	7	39
1	6	30
2	3	25
1	2	20
1	1	18
2	7	28

(20.) yards.	feet.	inches.
18	2	7
14	1	6
12	2	5
10	1	4
9	2	7
6	0	3

SQUARE, OR LAND MEASURE.

(21.) acres.	roods.	perches.
40	3	28
120	0	30
55	2	15
36	1	12
220	2	29
30	3	20

(22.) sq. yards.	feet.	inches.
28	5	30
15	4	15
20	0	6
19	3	42
17	2	139
15	8	120

CUBIC MEASURE.

(23.)

yards.	feet.	inches.
1	25	1720
	20	1640
1	26	40
	15	1727
1	19	1070
1	17	1063

(24.)

yards.	feet.	inches.
	25	167
1	26	76
1	20	482
	23	20
1	19	0
1	25	1643

BEER.

(25.)

hhds.	gals.	quarts.	pints.
5	62	3	1
3	56	2	0
7	40	1	1
1	27	3	0
4	61	0	1
2	60	3	0

WINE.

(26.)

gals.	qts.	pints.	gills.
50	3	1	3
49	2	0	1
40	1	0	2
39	2	1	2
27	3	0	0
46	3	1	3

DRY MEASURE.

(27.)

wey.	qr.	bushel.
1	4	7
	3	4
1	2	6
	4	2
1	3	6
1	2	5

(28.)

peck.	gall.	qr.
3	1	3
2	0	2
1	1	1
3	0	0
2	1	2
3	0	3

CLOTH MEASURE.

(29.)

Eng. ells.	qrs.	nls.
4	3	2
3	2	0
2	4	1
4	2	3
3	3	2
2	2	2

(30.)

yds.	qrs.	nls.
5	3	2
4	1	1
2	2	0
3	2	2
1	3	0
4	1	1

TIME.

(31.)

months.	weeks.	days.
12	3	6
11	2	5
9	1	3
7	3	2
6	2	1
10	0	6

(32.)

days.	hrs.	min.
6	23	50
4	19	45
3	20	30
2	17	6
5	6	51
4	3	2

ANGULAR MEASURE.

(33.)

deg.	min.	sec.
15	35	40
25	42	53
21	30	49
19	19	39
17	27	57
13	23	53

(34.)

signs.	deg.	min.	sec.
9	29	25	35
11	27	30	40
10	15	55	50
7	25	55	20
4	20	12	27
6	0	49	40

COAL MEASURE.

(35.)	chaldrons.	sacks.	bushels.	(36.)	strike.	bushels.	pecks.
	20	11	2		8	8	2
	14	9	1		2	6	1
	19	0	2		0	4	0
	17	10	0		1	2	3
	6	7	1		3	1	2
	15	8	2		2	7	1

WOOL WEIGHT.

(37.)	sacks.	weys.	tods.	(38.)	stones.	cloves.	pounds.
	11	1	5		19	1	5
	9	0	6		16	0	2
	6	1	3		13	1	6
	2	1	4		9	0	3
	1	0	5		17	1	1
	10	1	6		9	1	4
	8	0	3		13	1	1

COMPOUND SUBTRACTION.

By compound subtraction we find the *difference* of two numbers consisting of different names.

RULE.

Place the less number under the greater, so that the parts which are of the same name may stand under each other.

Begin with the lowest name, and take each number in the lower line from the number above it, and set down the remainder.

If the number in the lower line be greater than the number above it, subtract the lower one from as many of its name

as make one of the next higher, and add the remainder to the upper number; carry one to the next number in the lower line, and proceed thus to the end.

The method of proof is the same as in subtraction of simple numbers.

Example. From 243*l.* 13*s.* 7¼*d.* subtract 40*l.* 18*s.* 10½*d.*

£	*s.*	*d.*	
243	13	7¼	
40	18	10½	
202	14	8¾	difference.
243	13	7¼	proof.

The principle of this rule is the same as that of subtraction of simple numbers.

Examples.

(1.)

£	*s.*	*d.*
9	5	2
6	3	1

(2.)

£	*s.*	*d.*
1	3	4
	6	2

(3.)

£	*s.*	*d.*
1	5	6
1	13	4¼

(4.)

£	*s.*	*d.*
3	15	1½
1	12	8

(5.)

£	*s.*	*d.*
26	8	3½
17	6	9¾

(6.)

£	*s.*	*d.*
321	15	0
247	19	1¾

(7.)

£	*s.*	*d.*
427	4	2½
159	16	7¼

(8.)

£	*s.*	*d.*
1095	15	6¼
895	16	8½

(9.)

£	*s.*	*d.*
2150	18	5¾
1949	19	1¼

(10.)

£	s.	d.
1084	11	5
539	12	7

(11.)

£	s.	d.
6197	3	9
4593	8	$10\frac{3}{4}$

(12.)

£	s.	d.
21569	19	$1\frac{1}{2}$
20670	18	$2\frac{3}{4}$

TROY.

(13.)

oz.	dwt.	gr.
27	15	2
22	16	7

AVOIRDUPOIS.

(15.)

lb.	oz.	dr.
25	11	15
28	12	9

APOTHECARIES.

(17.)

℥	ʒ	℈	gr.
27	7	2	4
14	1	1	9

(14.)

lbs.	oz.	dwt.	gr.
52	1	7	2
39	7	15	8

(16.)

tons.	cwt.	qr.	lb.
21	15	1	21
18	9	0	27

(18.)

lb	℥	ʒ	℈
12	6	1	2
9	8	7	1

LONG.

(19.)

ft.	in.	bar.
26	9	2
19	10	1

SQUARE.

(21.)

a.	r.	p.
75	1	22
59	2	24

CUBIC OR SOLID.

(23.)

yds.	ft.	in.
87	15	920
65	18	1030

(20.)

lea.	m.	fur.	p.
72	1	4	23
59	2	7	27

(22.)

a.	r.	p.
325	2	6
289	3	9

(24.)

yds.	ft.	in.
212	25	450
109	27	189

LIQUID.

(25.)

tuns.	gal.	qts.
72	5	1
49	27	2

(26.)

tuns.	hhd.	gal.
78	1	15
39	3	18

DRY.

(27.)

qr.	bush.	pk.
69	1	1
58	3	3

(28.)

ch.	bush.	pk.
34	3	2
25	6	3

CLOTH.

(29.)

yds.	qrs.	nls.
36	2	1
18	3	2

(30.)

Fr. ells.	qrs.	nls.
34	1	3
18	5	2

TIME.

(31.)

h.	min.	sec.
76	28	13
62	49	19

(32.)

m.	w.	d.	h.
29	1	2	12
15	3	4	9

ANGULAR MEASURE.

(33.)

deg.	min.	sec.
85	42	15
79	49	12

(34.)

signs.	°	′	″
12	19	42	5
8	22	18	16

COAL MEASURE.

(35.)

ch.	sacks.	bush.
20	11	2
14	9	1

(36.)

strike.	bush.	pks.
3	8	2
2	6	1

WOOL WEIGHT.

(37.)

sacks.	wey.	tods.
11	1	5
9	0	6

(38.)

stones.	cloves.	lbs.
19	1	5
16	0	4

COMPOUND MULTIPLICATION.

By Compound Multiplication we find what a number, consisting of different names, will amount to, if repeated a certain number of times.

RULE.

Place the multiplier under the lowest name in the multiplicand.

Multiply each of the parts of the multiplicand, beginning with the lowest name, by the multiplier; and find, by division, how many units of the next higher name are contained in each product.

Set down the remainder, and carry the quotient to the next higher name. Proceed thus to the end.

Example. Multiply 452*l.* 11*s.* 10¾*d.* by 6.

£	*s.*	*d.*
452	11	10¾
		6
2715	11	4½ product, or answer.

This rule is the same in principle as that of multiplication of simple numbers.

When the multiplier is a composite number, neither of whose factors exceeds 12, multiply first by one of these factors, and then the product thus obtained by the other.

When the multiplier is not a composite number, take the composite number next less than it, and multiply by its factors; then add as many times the multiplicand as the composite number taken is less than the given multiplier.

As a method of proof, make the factors change places, and multiply again. Or, use other factors, and multiply again.

Example 1. Multiply 1 cwt. 2 qrs. 12 lbs. by 24.

cwt.	qr.	lb.		cwt.	qr.	lb.	
1	2	12		1	2	12	
		4				6	
6	1	20		9	2	16	
		6				4	
38	2	8	answer.	38	2	8	proof.

Example 2. Multiply 7*s.* 8½*d.* by 17.

£	*s.*	*d.*
	7	8½
		4
1	10	10
		4
6	3	4
	7	8½
6	11	0½ answer.

£	*s.*	*d.*
	7	8½
		2
	15	5
		8
6	3	4
	7	8½
6	11	0½ proof.

Examples.

1. Multiply 34*l.* 6*s.* 4¼*d.* by 2; also by 3, 4, 5, 6 and 7, performing each operation twice.

2. Multiply 420*l.* 10*s.* 8½*d.* by 8; also by 9, 10, 11 and 12, performing each operation twice.

3. Multiply 52*l.* 13*s.* 11¾*d.* by 14; also by 22, 42, 35, 56 and 108, and prove your work.

4. Multiply 109*l.* 17*s.* 10½*d.* by 75, 83, 97, 102, 123 and 124, and prove your work.

5. Multiply 3 cwt. 2 qrs. 15 lbs. by 3; also by 7, 16, 19, 54 and 89, and prove your work.

6. Multiply 15 lbs. troy, 3 oz. 14 dwts. 13 grs. by 4; also by 15, 23 and 72, and prove your work.

7. Multiply 26 yds. 3 qrs. 2 nls. by 5; also by 12, 17 and 44, and prove your work.

8. Multiply 1 sq. yd. 3 ft. 7 in. by 11; also by 20, 57 and 64, and prove your work.

9. 120 pairs of shoes at 4*s.* 6*d.* per pair.

10. 87 yards of silk at 7*s.* 5½*d.* per yard.

11. 100 lbs. of cotton at 2¾*d.* per lb.

12. If a gallon of oil costs 6*s.* 7¼*d.*, what is the cost of a tun?

13. A person buys 10 lbs. of tea at 4*s.* 3*d.* per lb., 18 lbs. of coffee at 1*s.* 3½*d.* per lb., 23 lbs. of moist sugar at 4¼*d.* per lb., 16 lbs. of loaf sugar at 7¾*d.* per lb., and 47 lbs. of candles at 6½*d.* per lb.; how much has he to pay?

14. A goes over a distance of 7 miles 5 furlongs 31 poles, 28 times in a month, B goes over 8 miles 3 furlongs 29 poles, 24 times in a month; which of them travels farthest, and by how much, in a year of 13 months?

COMPOUND DIVISION.

By Compound Division we learn to divide a number consisting of several names into any required number of equal parts.

RULE.

Place the divisor on the left of the dividend, with a curve between them.

Divide each name of the dividend, beginning at the left, by the divisor, and set down the quotient.

If, when dividing any name, there be a remainder, carry it to the next lower name as so many of that name as are equal to it, continue the division, and set down the quotient as before.

As a method of proof, multiply the quotient by the divisor.

Example 1. Divide 251*l.* 17*s.* 9¾*d.* into 4 equal parts.

	£	*s.*	*d.*	
4)	251	17	9¾	
quotient	62	19	5¼	– 3
			4	
proof	251	17	9¾	

The operation is practically performed thus; 4's in 25 six times and 1 over, set down 6 and carry 1; 4's in 11 twice and 3 over, set down 2 and carry 3; 3 pounds are 60 shillings, 60 and 17 are 77, 4's in 77 nineteen times and 1 over; 1 shilling is 12 pence, 12 and 9 are 21, 4's in 21 five times and 1 over; 1 penny is 4 farthings, 4 and 3 are 7, 4's in 7 once and 3 over, set down one farthing, and the remainder 3.

This example shews that a fourth part of 251*l.* 17*s.* 9¾*d.* is 62*l.* 19*s.* 5¼*d.* and 3-fourths of a farthing.

Example 2. Divide 543*l.* 11*s.* 10¾*d.* by 48.

	£	*s.*	*d.*		
6)	543	11	10¾		
8)	90	11	11¾	– 1	43 remainder.
quotient	11	6	5¾	– 7	
			8		
	90	11	11¾		
			6		
proof	543	11	10¾		

Example 3. Divide 471*l.* 12*s*, 7½*d.* by 37.

£	s.	d.	£	s.	d.	
37) 471	12	7½	(12	14	11	quotient
37					6	
101			76	9	6	
74					6	
27*			458	17	0	
20			12	14	11	
552					8½	
37			471	12	7½	proof
182						
148						
34						
12						
415						
37						
45						
37						
8						
4						

33 = 8½*d.* remainder.

Examples. To be proved by multiplication.

1. Divide 635*l.* 15*s.* 4¼*d.* by 2; also by 3, 4, 5 and 6.

2. Divide 730*l.* 11*s.* 3½*d.* by 7; also by 8, 9, 10, 11 and 12.

3. Divide 1349*l.* 13*s.* 2¾*d.* by 16; also by 27, 28, 33, 49 and 54.

* Here the remainder 27 pounds is multiplied by 20, to ascertain how many shillings are equal to it, and the 12 shillings in the dividend are added to them, thus making 552 shillings.

4. Divide 908*l.* 0*s.* 10*d.* by 41; also by 53, 71, 97, 109 and 7043.

5. Divide 50 tuns 1 pipe 1 hhd. 57 gals. by 8; also by 13, 20 and 59.

6. Divide 4 miles 2 fur. 31 poles by 11; also by 24, 31 and 42.

7. If a field of 5 acres is worth 481*l.* 16*s.*, what is the value of 1 acre?

8. If 27 quarters of corn are sold for 60*l.* 15*s.*, what is its price per quarter?

9. In a regiment of cavalry there are 625 horses, which consume 129 quarters 7 bushels 2 pecks of oats in a week; how much is the average weekly consumption for each horse?

10. There are 26 wedges of gold, of equal size, which weigh altogether 34 lbs. 3 oz. 11 dwts. 14 grs.; what is the weight of each wedge?

11. If a railway train goes 200 miles 3 furlongs in 14 hours, at what rate does it travel per hour, including stoppages?

12. A gentleman dies, leaving 2300*l.*, which, after payment of his debts, is to be distributed thus: one-half to his widow, one-third of the remainder to his son, and, what is then left, between his 3 daughters; what will each member of his family receive, his debts being 109*l.* 17*s.* 8*d.*, 54*l.* 10*s.*, 73*l.* 14*s.* 4*d.*, 16*l.* 8*s.*, 12*l.* 10*s.* 7*d.*, and 6*l.* 14*s.* 8*d.*?

To multiply any quantity, is to take it as many times as there are units in the multiplier.

Thus, if 6 acres be multiplied by 4, since there are 4 units in the multiplier, 6 acres are taken 4 times; consequently, 24 acres are taken.

Again, if 10 ounces be multiplied by $\frac{1}{2}$, since there is half an unit in the multiplier, 10 ounces are taken half a time; consequently, 5 ounces are taken.

Hence, to multiply by $\frac{1}{2}$ we divide by 2; and to multiply by $\frac{1}{4}$ we divide by 4.

Similarly, to multiply by $\frac{3}{4}$ we multiply by 3 and divide the product by 4; and by $\frac{5}{16}$ we multiply by 5, and divide the product by 16.

Example. 1. Find the value of $10\frac{1}{2}$ lbs. of butter at 1*s.* $2\frac{1}{2}$*d.* per lb.

s.	*d.*
1	$2\frac{1}{2}$
	$10\frac{1}{2}$
12	1
	$7\frac{1}{4}$
12	$8\frac{1}{4}$ answer.

Example 2. What is the value of $6\frac{5}{8}$ yards of satin, at 10*s.* 4*d.* per yard?

£	*s.*	*d.*
	10	4
		6
3	2	0
	6	$5\frac{1}{2}$
3	8	$5\frac{1}{2}$ answer.

£	*s.*	*d.*
	10	4
		5
8) 2	11	8
	6	$5\frac{1}{2}$

Miscellaneous Examples.

1. Find the amount of the following Mercer's bill.

	s.	*d.*		£	*s.*	*d.*
$6\frac{1}{4}$ yards of sarcenet, at	3	4	per yard			
$12\frac{1}{2}$ yards of satin, at	9	6	per yard			
$10\frac{3}{4}$ yards of velvet, at	24	5	per yard			
$15\frac{1}{8}$ yards of brocade, at	21	10	per yard			
$26\frac{1}{2}$ yards of lutestring, at	7	$1\frac{1}{2}$	per yard			
				£		

2. Repeat the tables of money, weights, and measures.

3. How many threes are there in 4 twelves?

4. Two persons start at the same time from the same place, to travel in the same direction, one goes $16\frac{1}{4}$ miles a day and the other $20\frac{1}{4}$; how far are they apart at the end of 8 days?

5. 120 sacks, at 1*l.* 4*s.* 6*d.* a sack.

6. How much money must be added to 271*l.* 11*s.* 5*d.* to make up the sum of 420*l.*?

7. If a pound of iron is worth 3¾*d.*, what is the value of a piece which weighs 2 cwt.?

8. How many years elapsed from the year 1627 to the year 1849?

9. A man received 1*l.* 14*s.* 2*d.* for working 10 days; at what rate per day was he paid?

10. A person bought 2 dozen oranges at 5 for a penny, and sold them at 4 for a penny; what did he gain or lose?

11. What is the difference between seventy thousand and forty, and six thousand three hundred and nine?

12. What is arithmetic? What is number? What is that number called which results from division? What is a product? What is a dividend, a multiplicand, a composite number? What is 10 + 7 equal to? Name the component parts of 25, 48, 100, and 72. How do you multiply by ¼? What is 19 — 11 equal to? What is the product of 12 and 7, what is their sum, and difference? Write 20 examples illustrating the use of the signs of addition, subtraction, multiplication, division, and equality. How many half-crowns are there in a pound? In 12*s.* 6*d*? In 7*s.* 6*d.*? In 17*s.* 6*d.*? In 4*l.*? In 2*l.* 10*s.*? How many sixpences in half a crown? In 4 half-crowns? In a guinea? How many inches in 2 yards? In 1½ foot? In 2¼ feet? How many pounds in 2 qrs.? In 1¾ cwt.? In 2 cwt.? If you knew the respective ages of a man and a woman, what rule should you apply to ascertain how much older one was than the other? If you knew a man's annual income, what rule should you apply to ascertain his weekly income? If you knew a place to be due south of London 47 miles, and another place due north of London 60 miles, what rule should you apply to ascertain the distance between the two places? If both were due south? If you knew the wages of one railway labourer, what rule should you apply to find the wages of 1000 railway labourers? If you knew the dates of two events, by what rule should you find the interval? If one happened before the birth of our Saviour and the other after it?

13. Add the following numbers together, 123456, 234561, 345612, 4561230, 561234, 612345, 54937, 8990 and 99.

14. *From* seven hundred and eighteen millions fourteen

thousand and fifty-seven, *take* ninety-seven millions eight hundred and four thousand seven hundred and sixteen.

15. Multiply 463098 by 7380; and 2778588 by 9867.

16. Divide 7954826 by 18; and 95642371 by 8790.

17. Add together, 42*l.* 13*s.* 4*d.*, 17*l.* 6*s.* 8¾*d.*, 90*l.* 9*s.* 8*d.*, and 21*l.* 12*s.* 4½*d.* Also 3 miles 5 fur. 137 yds. 9 in., 7 miles 7 fur. 77 yds. 7 in., 9 miles 6 fur. 203 yds. 6 in., 5 miles 4 fur. 156 yds. 2 in.

18. From 11 oz. 0 dwt. 0 grs., take 2 oz. 18 dwt. 22 grs.; from 37 ac. 2 rds. 29 pol., take 23 ac. 3 rds. 35 pol.; from 45 cub. yds. 24 cub. feet, 656 cub. in., take 12 cub. yds, 19 cub. feet, 999 cub in.

19. Multiply 42*l.* 10*s.* 9½*d.* by 88; and 5 days, 17 hrs. 39 min. 20 sec., by 120.

20. Divide 1062*l.* 10*s.* by 10000 and 576*l.* 3*s.* by 144.

21. Light travels at the rate of 192,500 miles per second; if a ray of light from the sun takes 490 seconds in reaching us, what is his distance from the earth?

22. Sound travels at the rate of 1130 feet per second: how far off is a thunder cloud when the sound is heard 7 seconds after the flash is seen, supposing the passage of light to be instantaneous?

REDUCTION.

Reduction is a method of changing the *name* of any quantity while its *value* continues unchanged.

Thus, the quantity 5 *pounds*, being changed into shillings, becomes 100 *shillings;* and this result may be produced by multiplying the 5 pounds by 20; that is, by as many shillings as are equal to a pound.

Again, the quantity, 48 inches, being changed into feet, becomes 4 feet; and this result may be produced by dividing the 48 inches by 12; that is, by as many inches as are equal to a foot.

RULE.

To reduce a quantity of any name to an equivalent quantity of a lower name.

Multiply by the number which shows how many of the lower name make one of the given name.

***Example* 1.** **Reduce 25 shillings to pence, and also to farthings.**

s.
25
12
300 pence.
4
1200 farthings.

Example 2. Reduce 10 yards 2 feet 9 inches, to inches.

yd.	ft.	in.
10	2	9
3		
32		
12		
393 inches.		

Here we say 3 times 10 are 30; 30 and 2 are 32 feet; 12 times 2 are 24, and 24 and 9 are 33; set down 3 and carry 3; 12 times 3 are 36; 36 and 3 are 39. Thus the 2 feet are added in with the feet, and the 9 inches with the inches.

Example 3. Reduce 48*l.* 17*s.* to shillings.

£	*s.*
48	17
20	
977 shillings.	

Here we say 0 is 0, and 17 is 17; set down 7 and carry 1; twice 8 are 16, 16 and 1 are 17, set down 7 and carry 1; twice 4 are 8, 8 and 1 are 9. Thus the 17*s.* are added in with the shillings.

RULE.

To reduce a quantity of any name to an equivalent quantity of a higher name.

Divide by the number which shows how many of the given quantity make one of the higher.

Example 1. Reduce 18880 pence to shillings and pounds.

12) 18880

2,0) 157,3–4

£78 13 4

Ans. 1573*s.* 4*d.*, and 78*l.* 13*s.* 4*d.*

Example 2. Reduce 157 yds. 3 qrs. to quarters, and also to English ells.

yds.	qr.
157	3
4	
5) 631	
126–1	

Ans. 631 qrs., and 126 English ells 1 qr.

Example 3. Reduce 14107 lbs. to tons.

28 { 7) 14107 lbs.
{ 4) 2015–2 } 23 lbs.
4) 503–3 }

2,0) 12,5–3 qrs.

6–5 cwt.

Ans. 6 tons 5 cwt. 3 qrs. 23 lbs.

The 23 lbs. are found by multiplying the remainder 3 by the divisor 7, and adding the remainder 2.

Examples to be proved by contrary process.

1. How many farthings are there in 23 shillings? Ans. 1104.
2. In 4762 farthings, how many pounds? Ans. 4*l.* 19*s.* 2½*d.*
3. In 162 lbs. 10 oz. 12 dwts. Troy, how many dwts? Ans. 39052.
4. Reduce 76452 grains Troy into ounces. Ans. 159 oz. 9 dwts. 6 grs.
5. Reduce 14 cwt. 2 qrs. 27 lbs. to pounds. Ans. 1651.
6. In 174682 oz. of lead, how many tons? Ans. 4 tons 17 cwt. 1 qr. 25 lbs. 10 oz.

7. In 47 cwt. 2 qrs. 27 lbs. 1 oz., how many drams?
Ans. 1368848.

8. In 46 quarters of wheat how many pecks? Ans. 1472.

9. Reduce 14168 pints of seed into bushels.
Ans. 221 bush. 1 pk. 1 gal.

10. In 578 English ells 3 qrs. 2 nls., how many French ells? Ans. 482 French ells 1 qr. 2 nls.

11. Reduce 468 yds. 2 qrs. 3½ nls. to Flemish ells.
Ans. 624 Flemish ells 0 qrs. 3½ nls.

12. In 2987 scruples how many pounds?
Ans. 10 lbs. 4 oz. 3 dr. 2 scru.

13. How many pints of wine in 16 tons 146 gals. 3 qts. 1 pint? Ans. 33431.

14. Reduce 786421 gills into pipes.
Ans. 195 pipes, 5 gals. 2 qts. 1¼ pts.

15. The area of a field is 6 acres 2 rds. 24 poles, how many perches does it contain? Ans. 1064.

16. In 126 acres 2 rds. 27 po., how many square links?
Ans. 1722695.

17. How many acres, square chains and links are there in 27921967 square links? Ans. 279 acres 2 chains 1967 lks.

18. In 697790512 square inches how many acres?
Ans. 108 acres 3 rds. 28 poles.

19. A agreed to give B 246 acres 1 rd. 37 po. in exchange for 23981265 square links. How many links did A lose by this agreement? Ans. 666860.

20. How many scores of coals are there in 2779672 pecks?
Ans. 813 score 8 chal. 2 bush.

21. In 4678200 seconds how many months?
Ans. 1 mo. 3 wks. 5 days 3½ hrs.

22. How many times does a clock tick between 12 P.M. on March 20th, and 21 min. past 11 A.M. on 15th December of the same year? Ans. 23282460.

23. Seven bins, each capable of containing 271687 cubic inches of corn are filled out of a large bin containing 82 cubic yds. 16 ft. 742 inches. How many cubic yds. of corn remain after filling them? Ans. 5 yds. 1 ft. 332 inches.

24. The wheel of a railway carriage makes 8640 revolutions in going from London to Slough (18 miles). What is the circumference of the wheel in feet? Ans. 11 feet.

25. The driving-wheel of a locomotive, 16 feet 8½ inches in circumference, makes 359 revolutions between two sta-

tions. What is the distance between them? Ans. 1 mile 239 yds. 1 ft. $8\frac{1}{2}$ inches.

26. The hind wheel of a coach is 14 ft. 3 inches in circumference, the fore wheel 10 feet 6 inches. How many revolutions will the latter make more than the former in travelling 66 miles 7 fur. 38 po. 5 yds.? Ans. $8865\frac{5}{7}$.

27. A person pays a debt of 460*l.* 16*s.* in sovereigns, half-sovereigns, crowns and shillings, of each an equal number. How many did he pay of each? Ans. 256.

28. What is the use of reduction? Should you multiply or divide in the following cases—by what numbers—and why? To reduce miles into feet? Pounds into pence? Inches into yards? Degrees into seconds? Tons into quarters? Gallons into tuns? Pounds into half-crowns? Pounds into four-pennies? Pecks into quarters? What is the difference between a farthing and 20*l.*? Between a penny and 1000*l.*? How many fifties are there in five twenties? How many pence in 30 shillings? How many tenpences in 60 shillings? What numbers will divide any number ending in 0? In 5? In any even number? Will an even number ever divide an odd one without remainder? Will an odd one ever divide an even one without remainder? What number will divide any number consisting of sixes and nines? Of sixes and ciphers? Of nines and threes? What number will divide every even number? What is a quotient? What is the product of 40 and 80? The sum? The difference? What is $43 + 16$? $14 + 7 - 21$? 16×6? $100 \div 5$? Repeat the tables of multiplication, money, weights and measures. In division will the quotient be the same if you previously divide both dividend and divisor by the same number? Will the difference of two quantities be changed if you increase them both by the addition of the same quantity to each? Illustrate this both by numbers and quantities.

RATIO.

Ratio is the relation which one quantity bears to another quantity of the same name with respect to magnitude.

If the numbers 12 and 3 be compared, it is obvious that the former is four times as great as the latter: hence the ratio of 12 to 3, which is expressed $12 : 3$, or $\frac{12}{3}$, is 4.

Similarly the ratio $10 : 2$ is 5, since 10 is 5 times 2.

Similarly the ratio 6 : 3 is 2, since 6 is twice 3.
" 8 : 6 is $\frac{1}{2}$, since 3 is one-half of 6.
and the ratio 2 : 3 is $\frac{2}{3}$, since 2 is two-thirds of 3.

PROPORTION.

Proportion is the equality of ratios.

Since the ratio of 12 : 3 is 4,

And the ratio of 8 : 2 is 4,

Therefore the ratio of 12 : 3 = the ratio of 8 : 2, or 12 : 3 :: 8 : 2 is a proportion, which is read thus, 12 *is to* 3 *as* 8 *is to* 2.

Similarly 15 : 5 :: 6 : 2 is a proportion, since the first ratio 15 : 5 = 3, and the second ratio 6 : 2 = 3.

And similarly of any other *equal* ratios.

In any proportion the product of the two mean terms is equal to the product of the two extreme terms.

Thus in the proportion 12 : 3 :: 8 : 2, $3 \times 8 = 12 \times 2$, and if each of these equal products be divided by 12, we shall have $\frac{3 \times 8}{12} = 2$, or the fourth term is equal to the product of the second and third divided by the first. The Rule of Three is an application of proportion. Thus, if 12 oranges cost 8 pence, what will 3 oranges cost? The answer is obviously 2 pence; for 3 oranges, being a quarter of 12 oranges, must cost a quarter of what 12 oranges cost, that is a quarter of 8 pence.

This question, stated by the Rule of Three, would stand thus

Oranges. Oranges. Pence.
12 : 3 :: 8 : the fourth term or answer,

which could be found as above, $\frac{3 \times 8}{12} = 2$ pence.

In the Rule of Three, there are three terms given to find the fourth.

RULE.

Consider which term is like the answer, and make it the third term.

If the answer will be greater than the third term, set the

greater of the other two in the middle, but if less, set the less in the middle. Set the other term first.

Reduce, when necessary, the first and second terms to the same name, and the third to the lowest name mentioned in it.

Multiply the second and third together, and divide by the first. The quotient will be the answer in the same name as the third term.

The operation may frequently be simplified by dividing the *first and second* or the *first and third* terms by a common divisor; since the value of a ratio is not changed by dividing both its terms by the same number.

This rule is applicable to interest, commission, brokerage, insurance, buying and selling stock, discount, partnership, barter, exchange, &c.

Example 1. A person buys 60 oranges for 4*s*. 2*d*., how many dozen can he buy at the same rate for 1*l*. 19*s*. 7*d*.?

Here the answer must be oranges; therefore we make 60 oranges the third term.

The answer will be greater than the third term, because more than 60 oranges can be bought for 1*l*. 19*s*. 7*d*.; therefore we set the greater term in the middle.

s.	*d*.		£	*s*.	*d*.		Oranges.
4	2	:	1	19	7	::	60
12			20				
50			39				
			12				
5			475				
1			95				
			6				

12)570 oranges.

47–6

Ans. 47 doz. 6 oranges, or 47½ doz.

In this example, after reducing the first and second terms to pence, we divide the *first and third* by 10, and then the *first and second* by 5, so that the statement ultimately stands

1 : 95 :: 6 : the fourth term;

we then multiply the second and third together, and since it is unnecessary to divide by 1, the product or 570 oranges is the fourth term.

We divide 570 by 12 to reduce them to dozens.

Example 2. How many yards of blue cloth, 3 qrs. wide, are equal in measure to 60 yards of black cloth 5 qrs. wide?

Here the answer must be length of cloth; therefore we make 60 yards long the third term.

The answer will be greater than the third term, because the blue cloth is narrower than the black; therefore we set the greater in the middle.

qrs. wide.		qrs. wide.		yds. long.
3	:	5	::	60
				5
				3)300
				Ans. 100 yds. long.

Example 3. If 5 men earn 100*l.* in 4 weeks, working 6 days a week, how much would 10 men earn in 3 weeks, working 5 days a week?

The answer must be money; therefore we make 100*l.* the third term. 10 men will earn more than 5 men; therefore we set the greater in the middle.

Men.	Men.	£	£
5 :	10 ::	100 :	$\frac{100 \times 10}{5}$ = sum 10 men

earn in 4 weeks.

Less money will be earned in 3 weeks than in 4; therefore we now set the less in the middle.

Weeks.	Weeks.	£	£
4 :	3 ::	$\frac{100 \times 10}{5}$:	$\frac{100 \times 10 \times 3}{5 \times 4}$ = sum

10 men earn in 3 weeks.

Less money will be earned by working 5 days a week than by working 6; therefore we again set the less in the middle.

Days.	Days.	£	£
6 :	5 ::	$\frac{100 \times 10 \times 3}{5 \times 4}$:	$\frac{100 \times 10 \times 3 \times 5}{5 \times 4 \times 6}$ =

sum 10 men earn in 3 weeks, working 5 days a week.

On inspecting this ultimate result, it is obvious that it consists of the product of the third and second terms divided by the product of the first terms: hence the above statements might have been set down thus

				£
Men.....	5	:	10	:: 100
Weeks...	4	:	8	
Days	6	:	5	

$$\frac{\cancel{100} \times \cancel{10} \times \cancel{8} \times \cancel{5}}{\cancel{5} \times \cancel{4} \times \cancel{6}} = 125l. \text{ Ans.}$$

The cancelling is performed on the same principle as dividing the terms of a ratio by a common divisor. Thus the two 5's cancel each other, the 3 divides the 6, the 2 divides the 10, the 4 divides the 100, after which there remain 25 and 5 to be multiplied together.

Example 4. If a journey can be performed in 12 days, travelling at the rate of 8 miles an hour for 6 hours each day, at what rate per hour must a person travel to go the same distance and back in 6 days, travelling 8 hours daily?

				Miles an hour.
Days.....	6	:	12	:: 8
Hours ...	8	:	6	
Journey..	1	:	2	

$$\frac{\cancel{8} \times 12 \times \cancel{6} \times 2}{\cancel{6} \times \cancel{8}} = 24 \text{ miles an hour. Ans.}$$

The answer must be miles per hour; therefore we make 8 miles the third term.

The double journey is to be done in 6 days instead of 12, hence he must go quicker; therefore we set the greater in the middle.

He will travel 8 hours a day instead of 6, hence he may go slower; therefore we set the less in the middle.

He has a double journey to perform instead of a single one, hence he must go quicker; therefore we set the greater in the middle.

Example 5. If an iron bar, 5 feet long, $4\frac{1}{2}$ inches broad, and $1\frac{1}{4}$ inches thick, weighs 45 lbs., how much will a bar of the same metal weigh, which is 14 feet long, 3 inches broad, and $2\frac{1}{4}$ inches thick?

					lbs.
Feet long.....	5	:	14	::	85
In. broad.....	$4\frac{1}{2}$	:	3		
In. thick	$1\frac{3}{4}$	:	$2\frac{1}{4}$		

Reducing the first and second terms in each statement we have

					lbs.
Feet long.....	5	:	14	::	85
Half in. broad.....	9	:	6		
Quarter in. thick...	7	:	9		

$$\frac{\overset{17}{\cancel{85}} \times \overset{2}{\cancel{14}} \times 6 \times \cancel{9}}{\cancel{9} \times \cancel{9} \times 7} = 204 \text{ lbs.} = 1 \text{ cwt. } 3 \text{ qrs. } 8 \text{ lbs.} \quad \text{Ans.}$$

Example 6. If 50*l*, principal gain 7*l*. 10*s*. interest in 9 months, what principal will gain 68*l*. interest in a year and a quarter?

£ *s.* int.		£ int.		£ prin.
7 10	:	68	::	50
yr.		m.		
$1\frac{1}{4}$	:	9		

Reducing the first and second terms in the first statement to half-sovereigns, and the first term of the second statement to months, we have

Half-sov....	15	:	136	:: 50
Months ...	15	:	9	

$$\frac{\overset{2}{\cancel{50}} \times 136 \times \cancel{9}}{\underset{\cancel{3}}{\cancel{15}} \times \underset{\cancel{3}}{\cancel{15}}} = 272l. \quad \text{Ans.}$$

In setting down the terms of each ratio in compound proportion, the student is strongly recommended to separate in his mind the conditions of the question which produce that ratio from the conditions which produce the others; thus in the last example, let him, in the first ratio, consider the condition of the *interest* without reference to the *time*, and in the second ratio that of the *time* without reference to the *interest*.

Again, in Ex. 5, let him reason thus, "A bar 5 feet long weighs 85 lbs., therefore a bar 14 feet long will weigh more."

"A bar $4\frac{1}{2}$ inches broad weighs 85 lbs., therefore a bar 3 inches broad will weigh less."

"A bar $1\frac{3}{4}$ inch thick weighs 85 lbs., therefore a bar $2\frac{1}{4}$ inches thick will weigh more."

Examples.

1. I bought 6 pounds of sugar for 4*s.*; how many pounds could I buy for 8*s.* *Ans. 12 pounds.

2. If 6 yards of ribbon cost 3*s.* 8*d.*, how much would 15 yards cost? Ans. 9*s.* 2*d.*

3. If 1 qr. 1 nl. of muslin cost $10\frac{3}{4}d.$, what must be given for 26 pieces, each containing 41 Fr. ells 4 qrs. $1\frac{3}{4}$ nl.?
Ans. 233*l.* 6*s.* $5\frac{3}{4}d.$

4. If a ton of coals cost 1*l.* 5*s.*, how much could be got for 10*l.*? Ans. 8 tons.

5. If 14 lbs. 10 oz. 16 dwts. 8 grs. of fine silver cost 67*l.* 2*s.* $4\frac{3}{4}d.$, what must be given for 1 oz.?
Ans. 7*s.* 6*d.* 3660 rem.

6. If a hundred cigars cost 1*l.* 5*s.*, how much would they cost apiece? Prove this by Compound Division. Ans. 3*d.*

7. Bought 14 Fl. ells 2 qrs. 1 nl. of fine satin at the rate of 12*s.* $4\frac{3}{4}d.$ per yard; I desire to know the price of the whole. Ans. 6*l.* 17*s.* $1\frac{1}{2}d.$

8. If a pair of stockings cost 3*s.* 4*d.*, how many dozen pairs can I buy for 43*l.* 5*s.*? Ans. 21 doz. $7\frac{1}{2}$ pairs.

9. If I buy 2 Fl. ells 2 qrs. 1 nl. of linen for 17*s.* $2\frac{3}{4}d.$, how many yards can I buy for 179*l.* 16*s.* 8*d.*?
Ans. 107 yds. 2 qrs. 2 nl. 186 rem.

10. If 1 lb. of sugar cost $5\frac{1}{2}d.$, what will 1 cwt. 2 qrs. come to at that rate? Ans. 3*l.* 17*s.* $8\frac{1}{2}d.$

11. If 1 cwt. of copper cost 3*l.* 16*s.* 8*d.*, what will 1 qr. 16 lbs. come to? Ans. 1*l.* 10*s.* $1\frac{1}{4}.\frac{5}{7}$

12. If 2 qrs. 14 lbs. of Congou tea cost 12*l.* 18*s.* $2\frac{3}{4}d.$, what must be given for 2 tons 13 cwt. 2 qrs. 27 lbs.?
Ans. 1110*l.* 4*s.* $0\frac{1}{4}\frac{3}{4}f.$

13. If 3 yds. 2 qrs. 2 nls. of velvet cost 16*l.* 10*s.* $8\frac{1}{2}d.$, what will 38 ells 4 qrs. $2\frac{1}{2}$ nls. come to?
Ans 221*l.* 18*s.* $10\frac{3}{4}d.$ 23 rem.

14. Bought 5 yds. 2 qrs. $3\frac{1}{2}$ nls. of cloth for 3*l.* 10*s.*; how many Eng. ells can I buy for 192*l.* 18*s.* $11\frac{1}{4}d.$?
Ans. 302 Eng. ells 1 qr. 1221. rem.

15. If, when working days are 12 hours long, a man can

* This question may be proved by taking the answer, and making a new question, thus: "If 12 lbs. cost 8*s.*, what will 6 lbs. cost?" or thus, "If 8*s.* be paid for 12 lbs., how many pounds can be got for 4*s.*?" A similar mode of proof will apply to any question in Proportion.

reap a field in 10 days, in how many days can he reap it when the day is 16 hours long? Ans. 7½ days.

16. 100 men could raise a parapet 8 feet high in 8 hours, but the work was required to be done in 2½ hours, how many men were required? Ans. 320 men.

17. If 500 boys would fill a square room, one of the sides being 20 feet, how many could a room 16 ft. by 10 ft. contain? Ans. 200 boys.

18. If 54 men can build a house in 90 days, how many men can do the same in 50 days? Ans. 97¼ men.

19. A courier makes a journey in 24 days by travelling 12 hours a day; how many days will he be in going the same journey, travelling 16 hours a day? Ans. 18 days.

20. If I was to lend a friend 600*l.* for 6 months, how long should he lend me 150*l.*? Ans. 2 yrs.

21. 100 men could dig a trench 10 feet wide and 180 feet long in 24 hours, but it was required to dig one of the same length 15 feet wide in the same time; how many men were required? Ans. 150 men.

22. The diameter of the hind wheel of a coach which travelled from London to Portsmouth was 4 feet, and it made 445 revolutions in a mile, what would be the diameter of the fore wheel which made 593⅓ revolutions? Ans. 3 feet.

23. A besieged town has a garrison of 1000 soldiers with provisions for only 3 months; how many must be sent out that the provisions may last 5 months? Ans. 400.

24. Standing upon level ground I observed that the shadow of a tower was 100 feet long, and the shadow of a pole 6 feet high, so placed that the extremity of its shadow exactly coincided with the extremity of the shadow of the tower, was 5 feet long; it is required to find the height of the tower? Ans. 120 feet.

25. A horse ran from Edmonton to Ware in 20 minutes at the rate of a mile in 3 minutes, but another horse ran it at the rate of 2½ miles in 3 minutes; how long was the second horse on the road? Ans. 16 minutes 40 seconds.

26. Two ships sailed together from Plymouth, one sailed at the rate of 5 miles an hour, and the other 4 on an average all the way, the first arrived at Calcutta in 103 days; in how many days did the other arrive? Ans. 128¾.

27. A person wishing to know his height saw that the length of his shadow at 12 o'clock was 4 feet 3 inches, but

the length of the shadow of a friend of his, whose height was 5 ft. 10 in., was 4 feet, required the person's height?

Ans. 6 ft. 2 in. 108 rem.

28. Two boats were crossing a river, one leaked one pint in every 2 minutes, the other one pint in every 5 minutes, the first, when it had gained the other side, contained $7\frac{1}{2}$ qts.; how much did the other contain, and what was the time of crossing? Ans. 3 qts.—and half an hour.

29. If a person walk 216 miles in 7 days of 16 hours each, in how many days of 12 hours each can he walk the same distance? Ans. 9 days 4 hours.

30. If 90 English degrees are equivalent to 100 French degrees, how many English degrees are equivalent to 654 French degrees? Ans. 588° 36′ Eng.

31. If a tradesman gains 1*s.* $4\frac{1}{2}$*d.* on an article which costs him 5*s.* 6*d.*, what does he gain on every 100*l.* he lays out?

Ans. 25*l.*

32. How much in length, that is 3 ft. 9 in. broad, will be equivalent to 37 ft. 9 in. in length, which is 7 ft. 6 in. broad?

Ans. 75 ft. 6 in.

33. The hands of a watch are exactly together at 12 o'clock, find the exact time when they will be together between the hours 1 and 2, 2 and 3, 7 and 8, and 8 and 9.

Ans. 5 m. $27\frac{3}{11}$ sec. past 1, 10 m. $54\frac{6}{11}$ sec. past 2, 21 m. $49\frac{1}{11}$ sec. before 8, and 16 m. $21\frac{9}{11}$ sec. before 9.

34. If 50*l.* principal gain 7*l.* 10*s.* interest in 9 months, what principal will gain 68*l.* interest in 15 months?

Ans. 272*l.*

35. If 40*l.* gain 4*l.* in 6 months at 20 per cent, what principal will gain 60*l.* in 16 months at 10 per cent.? Ans. 450*l.*

36. If 300*l.* be gained by lending 8700*l.* for 18 months, what will be gained by lending 580*l.* for 3 months?

Ans. 40*l.*

37. If 360 men can throw up a rampart of a citadel in 56 days of 8 hours each, the rampart being 400 feet long, 80 wide, and 20 deep, how many men would throw up another 500 feet long, 80 wide, and 50 deep, in 48 days of 10 hours each? Ans. 1050.

38. A bankrupt owes 3840*l.*, and his property amounts to 828*l.* only; what dividend will his creditors receive in the pound? Ans. 4*s.* 3$\frac{1}{4}$*d.*

39. What is the amount of 537*l.* 16*s.* 8*d.* in 4 years, at $2\frac{1}{2}$ per cent. per annum, simple interest?

Ans. 591*l.* 12*s.* 4*d.*

40. Find the commission or brokerage on 832*l.* at $2\frac{1}{2}$ per cent.? Ans. 20*l.* 16*s.*

41. An agent effects an insurance on property to the amount of 1850*l.*; what must be paid to him for his trouble at 3*s.* 9*d.* per cent.? Ans. 3*l.* 9*s.* $4\frac{1}{2}$*d.*

42. A person invests 2000*l.* in the 3 per cents. when they are at $88\frac{1}{2}$; what annual income is he entitled to?

Ans. 67*l.* 15*s.* $11\frac{11}{53}$*d.*

43. Find the discount of 355*l.* 5*s.* payable at the end of 4 months, at $4\frac{1}{2}$ per cent., simple interest? Ans. 5*l.* 5*s.*

44. Divide 1000*l.* among 3 persons, so that their shares may be as the numbers 3, 5, and 9?

Ans. 125*l.*, 312*l.* 10*s.*, and 562*l.* 10*s.*

45. A wine merchant mixes together 20 gallons of wine at 12*s.* a gallon, 25 gallons at 14*s.*, and 36 gallons at 16*s.*; what will be the value of a gallon of the mixture?

Ans. 14*s.* $4\frac{1}{2}$*d.* $\frac{2}{3}\frac{4}{1}$.

46. How many pounds Flemish are equivalent to 1050*l.* sterling, the course of exchange being 85 shillings Flemish for 1*l.* sterling? (20*s.* Flem. = 1*l.* Flem.).

Ans. 1487*l.* 10*s.* Flem.

47. If 16 plums are worth 12 pears, and 15 pears are worth ten apples, how many apples must be given for 72 plums? Ans. 36 apples.

48. When will the hands of a watch be together between 3 and 4 o'clock? Ans. $16\frac{4}{11}$ min. past 3.

49. When will the hands be together again if one makes a revolution in 10 hours and the other in 12? Ans. 60 hours.

50. When first after 8 o'clock is the minute hand directly opposite the hour hand? Ans. 8 h. $10\frac{10}{11}$ m.

51. A person sells a horse for 60*l.*, and thus loses 20 per cent. on the purchase-money, but he ought to have gained 10 per cent.; for how much was the horse sold below its proper value? Ans. 22*l.* 10*s.*

52. How many yards of shalloon, three quarters wide, will line a coat $2\frac{1}{2}$ yards long, yard wide? Ans. 3 yds. 2 qrs. 2 nls.

53. If a board be 9 inches broad, what must its length be to contain 4 square feet? Ans. 64 inches.

54. If the carriage of 60 cwt. cost 4*l*. 10*s*. for 20 miles, what weight ought to be carried 30 miles for 5*l*. 10*s*.?

Ans. 48 cwt. 3 qrs. 15 lbs.

55. What would be the purchase-money of an estate producing a rental of 3223*l*. at the rate of 2*l*. 15*s*. per cent.?

Ans. 117200*l*.

56. If 69 yards of carpet 3 qrs. wide cover a floor 8 yards 2 qrs. 2 nls. long, what is the width of the room?

Ans. 6 yds.

57. How many yards of cloth at 10*s*. a yard are worth 189 yards at 6*s*. 8*d*. a yard? Ans. 126 yds.

58. In what time will 1000*l*. amount to 5000*l*. at $4\frac{1}{2}$ per cent. per annum, simple interest? Ans. $88\frac{8}{9}$ years.

59. If in 12 days 126 acres are reaped by 7 men, in how many days will 16 men reap 72 acres? Ans. 3 days.

60. If 702 quarters of corn are consumed by 1878 soldiers in 336 days, how many quarters will an army of 22536 soldiers consume in 112 days? Ans. 2808 days.

61. If 27 men can do a piece of work in 14 days, working 10 hours a day, how many hours a day must 24 boys work in order to complete a similar piece of work in 45 days, the work of a boy being half that of a man? Ans. 7 hours.

62. How much stock in the 3 per cents. must I sell out to pay a debt of 550*l*., the price of the stock being $94\frac{1}{8}$, a commission of $\frac{1}{8}$ per cent. on the stock being paid to the broker?

Ans. 585*l*. 2*s*. $1\frac{2}{3}\frac{5}{7}d$.

63. How many grains does a sovereign weigh, the pound Troy of standard gold being coined into 46*l*. 14*s*. 6*d*.?

Ans. $123\frac{1}{6}\frac{7}{2}\frac{1}{3}$ grs.

64. If, when wine is 30*l*. per ton, 20*l*. worth will serve a ship's company of 336 men for 4 days at a pint a day each man, how long will 500*l*. worth serve a crew of 250 men at $1\frac{1}{2}$ pint a day each man, when the ton is worth but 24*l*.?

Ans. 112 days.

65. A garrison of 3600 men has just bread enough to allow 24 oz. a day to each man for 35 days; but a siege coming on, the garrison was reinforced, and then consisted of 4800 men; how many ounces of bread a day must each man be allowed, to hold out against the enemy 45 days?

Ans. 14 oz.

66. Three footmen, A, B, and C, start together, to go the

same way round an island 73 miles in circumference. A goes 5 miles a day, B 8, and C 10: when will they all be together again? Ans. in 73 days.

67. A hare, pursued by a dog, was 86 yards before him at starting; whilst the hare ran 5 yards, the dog ran 7: how far had the dog run when he caught the hare?

Ans. 301 yards.

68. A thief, pursued by a policeman, is 144 of his own steps ahead; now the thief takes four steps while the policeman takes 3, but the policeman goes as far in 2 steps as the thief goes in 3: how many steps must the policeman take to catch the thief? Ans. 864.

What is ratio? Is the value of a ratio altered by dividing or multiplying both its terms by a common divisor or multiplier? What is the value of the following ratios, 16 : 2, 20 : 5, 4 : 1, 9 : 12, 16 : 24?

Name other ratios equal to the above.

What is proportion? What is that property of proportion from which the Rule of Three is derived? Name four numbers in proportion. Name the fourth term in the following cases; 2 : 5 :: 10, 3 : 12 :: 1, 6 : 9 :: 2, 50 : 15 :: 100.

Which terms of a proportion may be divided by a common divisor? Would the fourth term remain the same if the second and third were transposed? In stating a question in the Rule of Three, which term should be like the answer? How do you determine where to place the other two terms? Which do you reduce to the same name? How must the other term be reduced? Is reduction of the terms sometimes unnecessary? What step do you take after performing the necessary reductions? In what name does the fourth term come in? What operation do you perform upon the remainder, if there is one? How would you prove a sum in proportion?

In compound proportion, which terms are multiplied together for a dividend, and which for a divisor? What caution must be observed in setting down the ratios in compound proportion?

Mention some of the principal rules in arithmetic to which proportion is applicable.

Miscellaneous Questions.

1. Reduce 2345678 drams to tons.

2. To how many persons may 59*l.* 9*s.* 6*d.* be distributed, giving 4*l.* 11*s.* 6*d.* to each?

3. A wedge of gold weighing 14 lb. 3 oz. 8 dwt. is valued at 514*l.* 4*s.*: what is the value of one ounce?

4. How many men will in 168 days reap a field which 108 men can reap in 266 days?

5. A general has 30,000 men, and his opponent 20,000. Supposing his opponent to kill or take 50,000, how many must he kill or take to keep the same advantage? Prove the operation.

6. A man buys oranges at the rate of 3 for 2*d.*; what must he give for 4 dozen at the same rate? If he sells them, and gains 16*d.*, how much is that per cent., and how much would he sell a hundred for, allowing 5 per cent. for cash?

7. If, at the ordinary rate of marching, soldiers take 75 steps in a minute, and at quick marching 108, how far will a regiment advance in 3 hours; $2\frac{1}{2}$ hours being at the first rate, and $\frac{1}{2}$ hour at the second rate, reckoning each step as 2 feet 8 inches long?

8. Reduce 5 miles 200 yards 3 inches, to inches.

9. Reduce 156423 drs. to cwts.

10. Reduce 200000 cubic inches into yards.

11. If 39 cwt. 1 qr. 11 lb. cost 59*l.* 6*s.* 6*d.*, required the cost of 13 cwt.

12. What quantity may be bought for 374*l.* 8*s.*, if the cost of 6 cwt. 2 qrs. is 27*l.* 14*s.* 8*d.*?

13. What is the income corresponding to an income-tax of 13*l.* 2*s.* 6*d.*, at the rate of 7*d.* in the pound.

14. If 12 horses plough 22 acres in 10 days, how many horses will plough 66 acres in 36 days?

15. What must a person's yearly income be, that he may spend 2*s.* 6*d.* a day, and lay by 20*l.* at the year's end?

16. If 300*l.* principal in one year gain 15*l.* interest, what will be the interest of 950*l.* for 9 months?

17. If in 6 days, of 10 hours each, 180 men can dig a trench 200 yards long, 3 wide and 2 deep, in how many days of

8 hours each can 100 men dig a trench 360 yards long, 4 wide, and 3 deep?

18. The hour and minute hand of a clock are exactly together at 12 o'clock: when are they next together?

19. What sum of money will amount to 138*l.* 2*s.* 6*d.* in 15 months at 5 per cent. per annum simple interest?

20. If 3 bushels of flour, at 3*s.* 5*d.* per bushel, 4 bushels, at 5*s.* 6*d.* per bushel, and 5 bushels at 4*s.* 8*d.* per bushel, be mixed together, what will be the value of a bushel of the mixture?

21. Three persons make a joint stock: A puts in 184*l.* 10*s.*, B 96*l.* 15*s.*, and C 76*l.* 5*s.*: they trade and gain 220*l.* 12*s.*: what is each man's share of that gain?

22. If tobacco be bought at 10*l.* 10*s.* per cwt., at what rate must it be retailed per lb. to gain 12 per cent.?

23. A merchant has 2000 yards of serge at $9\frac{1}{2}d.$ per yard, which he barters for canvas at $10\frac{1}{4}d.$ per yard: how many yards of canvas must he receive?

24. What will 2400*l.* stock in the 3 per cent. consols cost, when $84\frac{1}{8}l.$ will buy 100*l.* stock?

25. What must be paid for insurance of goods valued at 1200*l.* at $7\frac{5}{8}l.$ per cent.?

26. If 90 kreutzers are equal to a rix-dollar of Frankfort, and 4 kreutzers make 1 batzen, how many rix-dollars shall I receive for 500*l.* sterling, the course of exchange being $150\frac{3}{4}$ batzen per pound sterling?

TABLE OF THE ALIQUOT PARTS OF MONEY.

Of a Pound.								Of a Shilling.		
s.	*d.*		£	*s.*	*d.*		£	*d.*		*s.*
10	0	=	$\frac{1}{2}$	1	3	=	$\frac{1}{16}$	6	=	$\frac{1}{2}$
6	8	=	$\frac{1}{3}$	1	0	=	$\frac{1}{20}$	4	=	$\frac{1}{3}$
5	0	=	$\frac{1}{4}$		10	=	$\frac{1}{24}$	3	=	$\frac{1}{4}$
4	0	=	$\frac{1}{5}$		8	=	$\frac{1}{30}$	2	=	$\frac{1}{6}$
3	4	=	$\frac{1}{6}$		$7\frac{1}{2}$	=	$\frac{1}{32}$	$1\frac{1}{2}$	=	$\frac{1}{8}$
2	6	=	$\frac{1}{8}$		6	=	$\frac{1}{40}$	1	=	$\frac{1}{12}$
2	0	=	$\frac{1}{10}$		5	=	$\frac{1}{48}$	$\frac{3}{4}$	=	$\frac{1}{16}$
1	8	=	$\frac{1}{12}$		4	=	$\frac{1}{60}$	$\frac{1}{2}$	=	$\frac{1}{24}$
1	4	=	$\frac{1}{16}$		$3\frac{3}{4}$	=	$\frac{1}{64}$	$\frac{1}{4}$	=	$\frac{1}{48}$

TABLE OF THE ALIQUOT PARTS OF WEIGHTS AND MEASURES.

AVOIRDUPOIS WEIGHT.

Of a ton.

cwt.		
10	=	$\frac{1}{2}$
5	=	$\frac{1}{4}$
4	=	$\frac{1}{5}$
$2\frac{1}{2}$	=	$\frac{1}{8}$
2	=	$\frac{1}{10}$

Of a cwt.

qr.	lbs.		
2 or	56	=	$\frac{1}{2}$
1 or	28	=	$\frac{1}{4}$
	16	=	$\frac{1}{7}$
	14	=	$\frac{1}{8}$

Of $\frac{1}{2}$ cwt. or 56 lbs.

lbs.		
28	=	$\frac{1}{2}$
14	=	$\frac{1}{4}$
8	=	$\frac{1}{7}$
7	=	$\frac{1}{8}$

Of a $\frac{1}{4}$ cwt. or 28 lbs.

lbs.		
14	=	$\frac{1}{2}$
7	=	$\frac{1}{4}$
4	=	$\frac{1}{7}$
$3\frac{1}{2}$	=	$\frac{1}{8}$

Of a pound.

oz.		
8	=	$\frac{1}{2}$
4	=	$\frac{1}{4}$
2	=	$\frac{1}{8}$

TROY WEIGHT.

Of a pound.

oz.		
6	=	$\frac{1}{2}$
4	=	$\frac{1}{3}$
3	=	$\frac{1}{4}$
2	=	$\frac{1}{6}$

Of an ounce.

dwts.		
10	=	$\frac{1}{2}$
5	=	$\frac{1}{4}$
4	=	$\frac{1}{5}$
2	=	$\frac{1}{10}$

Of a dwt.

grs.		
12	=	$\frac{1}{2}$
8	=	$\frac{1}{3}$
6	=	$\frac{1}{4}$
4	=	$\frac{1}{6}$
3	=	$\frac{1}{8}$
2	=	$\frac{1}{12}$

LAND MEASURE.

Of an acre.

r.	p.		
2	0	=	$\frac{1}{2}$
1	0	=	$\frac{1}{4}$
	32	=	$\frac{1}{5}$
	20	=	$\frac{1}{8}$
	16	=	$\frac{1}{10}$
	8	=	$\frac{1}{20}$

CLOTH MEASURE.

Of a yard.

qr.	n.		
2	0	=	$\frac{1}{2}$
1	0	=	$\frac{1}{4}$
	2	=	$\frac{1}{8}$
	1	=	$\frac{1}{16}$

Of an English ell.

qr.	n.		
2	2	=	$\frac{1}{2}$
1	1	=	$\frac{1}{4}$
1	0	=	$\frac{1}{5}$
	2	=	$\frac{1}{10}$
	1	=	$\frac{1}{20}$

Of a Flemish ell.

qr.	n.		
1	2	=	$\frac{1}{2}$
1	0	=	$\frac{1}{3}$
	3	=	$\frac{1}{4}$
	2	=	$\frac{1}{6}$
	1	=	$\frac{1}{12}$

Of a French ell.

qr.	n.		
3	0	=	$\frac{1}{2}$
2	0	=	$\frac{1}{3}$
1	2	=	$\frac{1}{4}$
1	0	=	$\frac{1}{6}$
	3	=	$\frac{1}{8}$
	2	=	$\frac{1}{12}$
	1	=	$\frac{1}{24}$

PRACTICE.

By Practice we find the value of any quantity when the price of a unit of it is given.

RULES.

If the price be an odd number of shillings, multiply by the shillings, and divide the product by 20.

If the price be an even number of shillings, multiply by half of it, doubling the units figure of the product for shillings; the rest will be pounds*.

If the quantity consist of different names, as cwts., qrs., and lbs., multiply by the highest name, and take aliquot parts of the rest.

A few examples will illustrate the general mode of proceeding.

Example 1. Find the entire cost of the following articles: 752 yards tape at $\frac{1}{2}d.$ a yard, 675 yards at $2\frac{1}{2}d.$ a yard, 709 ells muslin at $6s.$ $4\frac{1}{2}d.$ per ell, 1075 yards ribbon at $4s.$ $7\frac{1}{4}d.$ per yard, 173 yards velvet at $2l.$ $5s.$ per yard, and 125 ells of stuff at $18s.$ per ell.

$\frac{1}{2}$	$\frac{1}{2}$	752	= value at	$1d.$
$\frac{1}{4}$	$\frac{1}{2}$	376	= ...	$\frac{1}{2}d.$
		188	= ...	$\frac{1}{4}d.$
		12)564	= ...	$\frac{3}{4}d.$
		20) 47		
		£2 7$s.$		

$2d.$	$\frac{1}{6}$	675	= value at		$1s.$	
$\frac{1}{2}$	$\frac{1}{4}$	112	6	= ...	$2d.$	
		28	$1\frac{1}{2}$	= ...	$\frac{1}{2}d.$	
		20)140	$7\frac{1}{2}$	= ...	$2\frac{1}{2}d.$	
		£7	0	$7\frac{1}{2}$		

* This is the same thing, in effect, as to multiply by the shillings and divide by 20.

5*s.*	$\frac{1}{4}$	709			=	value at £1		
1*s.* 3*d.*	$\frac{1}{4}$	177	5	0	=	... 5*s.* 0*d.*		
$1\frac{1}{2}d.$	$\frac{1}{10}$	44	6	3	=	... 1*s.* 3*d.*		
		4	8	$7\frac{1}{2}$	=	...		$1\frac{1}{2}d.$
		£225	19	$10\frac{1}{2}$	=	...	6*s.*	$4\frac{1}{2}d.$

4*s.*	$\frac{1}{5}$	1075			=	value at £1		
6*d.*	$\frac{1}{8}$	215			=	... 4*s.*		
1*d.*	$\frac{1}{24}$	26	17	6	=	...	6*d.*	
$\frac{1}{4}d.$	$\frac{1}{96}$	4	9	7	=	...	1*d.*	
		1	2	$4\frac{3}{4}$	=	...		$\frac{1}{4}d.$
		£247	9	$5\frac{3}{4}$	=	...	4*s.*	$7\frac{1}{4}d.$

5*s.*	$\frac{1}{4}$	173	=	value at £1			125	
		2					9	
		346	=	... £2				
		43	5	=	...	5*s.*	1125	
		£389	5*s.*	=	...	£2 5*s.*	£112	10*s.*

	£	*s.*	*d.*
752 yards tape, at $\frac{1}{4}d.$ per yard	2	7	0
675 yards ditto, at $2\frac{1}{2}d.$	7	0	$7\frac{1}{2}$
709 ells muslin, at 6*s.* $4\frac{1}{2}d.$ per ell.....	225	19	$10\frac{1}{2}$
1075 yards ribbon, at 4*s.* $7\frac{1}{4}d.$ per yard	247	9	$5\frac{3}{4}$
173 yards velvet, at 2*l.* 5*s.* per yard	389	5	0
125 yards stuff, at 18*s.* per yard.....	112	10	0
Ans.	£984	11	$11\frac{3}{4}$

Example 2. What is the entire cost of the following miscellaneous articles?

	£	s.	d.	
172⅜ yards of cotton, at 1s. 1½d. per yard...	9	13	11	·25
12 cwt. 3 qrs. of sugar, at 2l. 4s. per cwt....	28	1	0	
473½ tons of coal, at 1l. 17s. 6d. per ton...	817	16	3	
50 oz. 16 dwt. of silver at 6s. 3¼d. per oz.	15	18	6½	·80
247 Fl. ells of cloth at 19s. 11d. per Fl. ell	245	19	5	
Ans.	£1187	9	1¾	·05

Operation.

1½	⅛	172			= value at 1s.			s.	d.		
		21	6		= ,, 1½d.			1	1½		
			5	–	·25* value of ⅜ yd.				3		
2,0)		19,3	11	–	·25			8) 3	4½		
		£9	13	11	– ·25				5	–	.25

		£	s.	d.			
2 qr.	½	2	4	0	= value of 1 cwt.		
				12			
		26	8	0	= ... 12 cwt.		
1 qr.	½	1	2	0	= ...	2 qr.	
			11	0	= ...	1 qr.	
		£28	1	0	= value of 12 cwt. 3 qr.		

10s.	½	473			= value at £1		£	s.	d.	
5s.	½	236	10	0	= ... 10s.	2)	1	17	6	
2s. 6d.	½	118	5	0	= ... 5s.					
		59	2	6	= ... 2s. 6d.			18	9	
			18	9	= ... value of ½ ton.					
		£887	16	3						

* These decimal parts of a farthing are obtained by borrowing ciphers, and continuing the division. The operation is explained in Decimals.

		s.	d.
10 dwt.	½	6	3¼
			5
		1 11	4¼
			10
5 dwt.	¼	15 13	6½
1 dwt.	⅕	3	1½·5
		1	6¾·25
			3¾·05
		£15 18	6½·80

£	s.	d.				d.
247	0	0	= value at £1		12)	247
1	0	7	= ... 1*d.*			20 7
£245	19	5	= value at 19*s.* 11*d.*			

Work out both the above examples by Proportion.

Examples.

1. 3846 at ¼*d.* Ans. £4 0*s.* 1½*d.*
2. 7980 at ¾*d.* Ans. £24 8*s.* 9*d.*
3. 874 at 2*d.* Ans. £7 5*s.* 8*d.*
4. 8146 at 3*d.* Ans. £101 16*s.* 6*d.*
5. 4587 at 6¾*d.* Ans. £129 0*s.* 2¼*d.*
6. 1546 at 10½*d.* Ans. £67 12*s.* 9*d.*
7. 8418 at 11¼*d.* Ans. £394 11*s.* 10½*d.*
8. 9836 at 11¾*d.* Ans. £481 11*s.* 1*d.*
9. 568 at 1*s.* 4½*d.* Ans. £39 1*s.*
10. 7189 at 1*s.* 8¾*d.* Ans. £621 10*s.* 11¾*d.*
11. 8710 at 1*s.* 5¾*d.* Ans. £644 3*s.* 6½*d*
12. 7481 at 1*s.* 11¼*d.* Ans. £740 6*s.* 1¾*d.*
13. 3508 at 8*s.* Ans. £1403 4*s.*
14. 6148 at 12*s.* Ans. £3688 16*s.*
15. 8471 at 11*s.* Ans. £4659 1*s.*
16. 9817 at 19*s.* Ans. £9326 3*s.*
17. 4108 at 9*s.* 6*d.* Ans. £1951 6*s.*
18. 7546 at 11*s.* 8*d.* Ans. £4401 16*s.* 8*d.*
19. 1106 at 15*s.* 10½*d.* Ans. £877 17*s.* 9*d.*
20. 5683 at 18*s.* 7½*d.* Ans. £5292 5*s.* 10½*d.*

21. 4187 at £3 11*s.* 6½*d.* Ans. £14977 14*s.* 11½*d.*
22. 2179 at £4 18*s.* 5½*d.* Ans. £10727 0*s.* 8½*d.*
23. 3145 at £14 17*s.* 9¼*d.* Ans. £46824 9*s.* 3¼*d.*
24. 4627 at £18 14*s.* 5*d.* Ans. £86621 5*s.* 11*d.*
25. 234½ lbs. at 3*s.* 4*d.* per lb. Ans. £39 1*s.* 8*d.*
26. 5896¾ at 11*s.* 6*d.* Ans. £3390 12*s.* 7½*d.*
27. 3278⅗ at £2 12*s.* 6*d.* Ans. £8606 6*s.* 6*d.*
28. 9876 1/16 at £3 18*s.* 9*d.* Ans. £38889 9*s.* 1½*d.* ¾*f.*
29. 3 cwt. 2 qr. 14 lb. of sugar, at £3 12*s.* 6*d.* per cwt.
Ans. £13 2*s.* 9¾*d.*
30. 72 yds. 3 qr. 2 nl. of cotton, at 2*s.* 3*d.* per yard.
Ans. £8 3*s.* 11½*d.*
31. 500 qrs. 4 bush. of oats, at £1 8*s.* 11*d.* per quarter.
Ans. £723 12*s.* 9½*d.*
32. 59 tons 15 cwt. 3 qr. 18 lb. of lead, at £53 17*s.* 6*d.* per ton. Ans. £3224 3*s.* 6½*d.*
33. 12 barrels coffee, each 125 lb., at 138*s.* per cwt.
Ans. £92 8*s.* 2½*d.*
34. 16 parcels cotton wool, each 3 qr. 2 lb., at 18*d.* per lb. Ans. £106 1*s.* 4*d.*

Ex. 1. What is the amount of the following draper's bill?

	£	*s.*	*d.*
371 yds. of tape, at ¼*d.* per yd.			
425 yds. of ribbon, at ½*d.* per yd.			
5714 yds of braid, at ¾*d.* per yd.			
506 balls of cotton, at ½*d.* per ball			
1896 knots of silk, at ¾*d.* per knot			
£			

Ex. 2. What is the amount of the following fruiterer's bill?

	£	*s.*	*d.*
19 qts. of currants, at 1*d.* per qt.			
10 qts. of gooseberries, at 1½*d.* per qt.			
100 oranges, at 1¼*d.* each			
74 qts. of strawberries, at 3*d.* per qt.....			
49 lbs. of cherries, at 4½*d.* per lb.			
31 lbs. of plums, at 6¾*d.* per lb.			
£			

Ex. 3. What is the amount of the following butcher's bill?

	£	*s.*	*d.*
A loin of lamb, weight 7¼ lb., at 6½*d.* per lb....			
A piece of veal, weight 16 lb., at 4¼*d.* per lb...			
A buttock of beef, weight 20¾ lb., at 4½*d.* per lb.			
A leg of pork, weight 12 lb., at 5¼*d.* per lb....			
A leg of mutton, weight 13 lb., at 4¾*d.* per lb.			
£			

Ex. 4. What is the amount of the following haberdasher's bill?

	£	*s.*	*d.*
240 collars at 7*d.* each			
429 pairs of thread gloves, at 10*d.* per pair ...			
106 pairs of socks, at 11½*d.* per pair			
60 napkins, at 10¾*d.* each			
24 pairs of stockings, at 11¼*d.* per pair			
£			

Ex. 5. What is the amount of the following cheesemonger's bill?

	£	*s.*	*d.*
1 cwt. of Belfast butter at 13*d.* per lb.			
28 Stilton cheeses, weight 227lb., at 14¼*d.* per lb.			
320 cream cheeses, wt. 260lb., at 13½*d.* per lb.			
8 dozen eggs, at 15*d.* per dozen.....			
4 hams, weight 48 lb., at 8½*d.* per lb.....			
£			

Ex. 6. What is the amount of the following grocer's bill?

	£	*s.*	*d.*
257½ lb. of green tea, at 4*s.* 9*d.* per lb.			
319 lb. of good hyson, at 5*s.* 9½*d.* per lb.			
418¼ lb. of bohea, at 6*s.* 9*d.* per lb.			
619 lb. of finest bloom, at 6*s.* 8¾*d.* per lb			
510 lb. of black, at 4*s.* 6*d.* per lb.			
320 lb. of coffee, at 1*s.* 3½*d.* per lb.....			
£			

Ex. 7. What is the amount of the following wine merchant's bill?

	£	*s.*	*d.*
473⅛ galls. of British spirits at 14*s.* 7½*d.* per gall.			
41 galls. of Maidstone gin at 14*s.* 6*d.* per gall.			
207 galls. of rum at 18*s.* 9¾*d.* per gall.			
119 galls. of Cognac brandy at 30*s.* 0½*d.* per gall.			
1 hhd. of old port at 16*s.* 9*d.* per gall.			
2 hhd. of claret at 18*s.* 11*d.* per gall			
£			

Ex. 8. What is the amount of the following goldsmith's bill?

	£	*s.*	*d.*
Silver cup, weight 8 oz. 11 dwt., at 6*s.* 4*d.* per oz.			
Silver salver, weight 10 oz. 5 dwt., at 5*s.* per oz.			
Gold cup, weight 8 oz. 4 dwt. 20 grs., at £3 per oz.			
Silver urn, weight 6 lb. 10 oz., at 5*s.* 9¼*d.* per oz.			
Gold salver, weight 1 lb. 4 oz. 12 dwt., at £4 per oz.			
£			

Ex. 9. What is the amount of the following stationer's bill?

	£	*s.*	*d.*
10 thousand old pens at 2*s.* 6*d.* per hundred...			
A ream of paper at 2*s.* per quire.....			
245 dozen of copy books at 1*s.* 2*d.* each.....			
146 dozen lead pencils at 6*s.* per dozen.....			
44 dozen bottles of ink at 1*s.* 1*d.* per bottle ...			
£			

Ex. 10. What is the amount of the following corn merchant's bill?

	£	*s.*	*d.*
Oats, 245 quarters, at 37*s.* per quarter.....			
Wheat, 260 quarters, at 48*s.* per quarter			
Barley, 670 quarters, at 36*s.* per quarter			
Peas, 127 bushels, at 3*s.* 7*d.* per bushel			
Malt, 46 quarters, at 43*s.* per quarter			
£			

Ex. 11. What is the amount of the following grocer's bill?

	£	*s.*	*d.*
16 cwt. 1 qr. 19 lb. of sugar at £3 10*s.* per cwt.			
21 cwt. 2 qr. 17¾ lb. of sugar at £4 4*s.* per cwt.			
19 cwt. 1 qr. 21¼ lb. of raisins at £2 7*s.* per cwt.			
11 cwt. 3 qr. 14½ lb. of currants at £2 10*s.* 8*d.* per cwt.			
66 cwt. 2 qr. 6¾ lb. of dates at £3 5*s.* per cwt...			
5 cwt. 4 qr. 7½ lb. of raisins at £2 4*s.* per cwt..			
£			

Ex. 12. What is the amount of the following draper's bill?

	£	*s.*	*d.*
676½ yds. of broad cloth at 15*s.* 7½*d.* per yd....			
234 Eng. ells 4 qr. 2 nl. of tweed at 16*s.* 4¾*d.* per Eng. ell.....			
67 Fr. ell 2 qr. 1 nl. of French cambric at 8*s.* 9*d.* per Fr. ell.....			
126 Fl. ell 2 qr. 3 nl. of cloth at 14*s.* 6¾*d.* per Fl. ell.....			
766 yds. of Irish sheeting at 4*s.* 6*d.* per yd.....			
127 Eng. ells 4 qr. 3 nl. of damask at 10*s.* 6½*d.* per ell			
£			

Ex. 13. Thomas Haines, Esq.

Bought of James Kingston.

Sept. 6, 1849.	£	*s.*	*d.*
Tares, 108 bushels, at 2*s.* 7*d.* per bushel			
Peas, 127 bushels 3 pecks, at 1*s.* 10½*d.* per bushel			
Malt, 46¼ quarters, at £1 14*s.* 4*d.* per quarter...			
Oats, 204⅛ quarters, at 18*s.* 6*d.* per quarter.....			
Beans, 17⅞ quarters, at £1 17*s.* 6*d.* per quarter			
	£327	7	6

Ex. 14. Dublin, July 10, 1849.

Mr. Simpkin,

Bought of Robert Harris.

cwt.	qr.	lb.		£	s.	d.		£	s.	d.
14	1	19	tobacco at	4	17	2	per cwt.			
17	2	17	snuff at	5	19	4	.....			
18	3	16	tobacco at	3	10	8	.....			
9	0	15	sugar at	2	12	6	.....			
	3	10	soap at	2	17	4	.....			
49	1	9	molasses at	1	16	4	.....			
								£358	2	7

Ex. 15. Mr. Joseph Lewis,

Bought of Samuel Downshire.

June. 10, 1849.

Six parcels muslin, viz.,

	ell.	qr.	nl.		s.	d.		£	s.	d.
No 1.	24	2	1	at	6	9	per ell.			
2.	27	1	3	,,	7	2	.....			
3.	21	0	2	,,	8	4	.....			
4.	34	1	1	,,	5	3	.....			
5.	19	2	2	,,	7	9	.....			
6.	27	1	3	,,	7	2½	.....			
								£40	5	8½

Ex. 16. Mr. Henry Swainson

To White, Freelove, and Co. Dr.

Manchester, 1849.	£	s.	d.
Feb. 17. 4½ pieces muslin, each 37¼ yds., at 10*s.* 7½*d.* per Eng. ell.....			
,, 25. 7¾ pieces chintz, each 47½ yds., at 4*s.* 6½*d.* per Eng. ell.....			
Mar. 7. 4⅛ pieces Holland, each 25½ Fl. ells, at 3*s.* 10*d.* per yd.....			
Carried forward			

		£	s.	d.
	Brought forward			
Mar. 11.	$10\frac{3}{8}$ pieces French serge, each $19\frac{1}{2}$ ells Fr. at 2*s.* $9\frac{1}{4}$*d.* per yd.....			
„ 30.	$1749\frac{1}{8}$ yds. Kendal cottons, at $9\frac{1}{8}$*d.* per ell Fl.....			
„ „	$947\frac{1}{4}$ yds. Manchester stuff, at $10\frac{3}{4}$*d.* per ell Fl.....			
		£339	17	$1\frac{1}{4}$

SECTION II.

FRACTIONS.

A FRACTION is a part, or a number of parts, of an unit; as $\frac{1}{4}$ £, one-fourth of 1 pound; $\frac{3}{5}$ mile, three-fifths of 1 mile. The lower number represents the number of equal parts the unit is divided into, and is called the *denominator*.

The upper number shows how many of those parts are taken to form the fraction, and is called the *numerator*.

Thus, if an orange be cut into 6 equal parts, and one of them be taken, $\frac{1}{6}$ of the orange will be taken, the denominator of the fraction being 6, and the numerator 1. If 2 of the parts be taken, $\frac{2}{6}$, that is, $\frac{1}{3}$ of the orange will be taken. If 3 of the parts be taken, $\frac{3}{6}$ or $\frac{1}{2}$ of the orange will be taken. If 4 of the parts be taken, $\frac{4}{6}$ or $\frac{2}{3}$ of the orange will be taken. If 5 of the parts be taken, $\frac{5}{6}$ of the orange will be taken. If 6 of the parts be taken, $\frac{6}{6}$, that is, the whole of the orange, will be taken.

In considering this case, it is evident that if the denominator remains constant the fraction will be larger or smaller, according as the numerator is larger or smaller: that if the numerator is constant, the fraction is larger when the denominator is smaller; that when the denominator and numerator are equal, the fraction is = 1; and that the value of a fraction is not changed by multiplying or dividing both its terms by the same number.

A *proper* fraction is one whose numerator is less than its denominator; as $\frac{3}{11}$, $\frac{12}{35}$.

An *improper* fraction is one whose numerator is either equal to or greater than its denominator; as $\frac{5}{5}$, $\frac{11}{3}$.

A *mixed number* consists of a whole number and a fraction; as $1\frac{3}{4}$, $10\frac{2}{7}$.

A *compound* fraction is a fraction of a fraction; as $\frac{2}{5}$ of $\frac{4}{9}$. It may be considered as the product of two or more single fractions.

A *complex fraction* is of the form $\frac{2\frac{3}{7}}{10}$, $\frac{5}{7\frac{1}{2}}$, $\frac{\frac{2}{3}}{\frac{4}{11}}$, $\frac{4\frac{1}{2}}{\frac{3}{8}}$, &c.

Both proper and improper fractions are called *simple*. A fraction may be considered like a ratio, the numerator being the dividend, and the denominator the divisor: thus $\frac{12}{6}$ of an orange is the same thing as 2 oranges.

MULTIPLICATION AND DIVISION OF FRACTIONS.

Twice 1 quarter is evidently 2 quarters, that is, $2 \times \frac{1}{4} = \frac{2}{4}$ or $\frac{1}{2}$.

Hence, to multiply a fraction by a number, we multiply its numerator, or divide its denominator, by that number.

Half 1 quarter is evidently 1 eighth, that is $\frac{1}{2}$ of $\frac{1}{4} = \frac{1}{8} = \frac{1}{4} \div 2$.

Half 2 eighths is evidently 1 eighth, that is, $\frac{1}{2}$ of $\frac{2}{8} = \frac{1}{8} = \frac{1}{4} \div 2$.

Hence, to divide a fraction by a number, we multiply its denominator, or divide its numerator by that number.

A third of $\frac{3}{4}$ is consequently $\frac{3}{12}$
therefore 2 thirds of $\frac{3}{4}$ must be $\frac{6}{12}$
} that is, $\frac{3}{4} \times \frac{2}{3} = \frac{6}{12}$.

Hence, to multiply fractions together, we multiply their numerators together for a numerator, and their denominators together for a denominator.

Hence also, since division is the reverse of multiplication, to divide one fraction by another, we invert the divisor, and proceed as in multiplication.

Ex. 1. Multiply $\frac{2}{5}$ by 6; $\frac{3}{8}$ by $\frac{6}{7}$; 15 by $\frac{4}{9}$; and $\frac{2}{9}$ by $\frac{12}{8}$.

$$\frac{2}{5} \times 6 = \frac{12}{5}; \quad \frac{3}{\cancel{8}_{4}} \times \frac{\cancel{6}^{3}}{7} = \frac{9}{28}; \quad \frac{\cancel{15}^{5}}{1} \times \frac{4}{\cancel{9}_{3}} = \frac{20}{3}; \quad \frac{\cancel{2}}{\cancel{9}_{3}} \times \frac{\cancel{12}^{4}}{\cancel{8}_{\cancel{4}}} = \frac{2}{3}.$$

Ex. 2. Multiply $\frac{4}{17}$ by 2; $\frac{1}{24}$ by $\frac{9}{2}$; 12 by $\frac{5}{8}$; and $\frac{3}{8}$ by $\frac{12}{15}$.

Ex. 3. Divide $\frac{4}{5}$ by 8; $\frac{6}{11}$ by $\frac{5}{44}$; 18 by $\frac{12}{13}$; and $\frac{3}{8}$ by $\frac{15}{12}$.

$$\frac{4}{5} \div 8 = \frac{\cancel{4}}{5} \times \frac{1}{\cancel{8}_{2}} = \frac{1}{10}; \quad \frac{6}{11} \div \frac{5}{44} = \frac{6}{\cancel{11}} \times \frac{\cancel{44}^{4}}{5} = \frac{24}{5};$$

$$18 \div \frac{12}{13} = \frac{\cancel{18}^{3}}{1} \times \frac{13}{\cancel{12}_{2}} = \frac{39}{2}; \quad \frac{3}{8} \div \frac{15}{12} = \frac{\cancel{3}}{\cancel{8}} \times \frac{\cancel{12}^{3}}{\cancel{15}_{5}} = \frac{3}{5}.$$

Ex. 4. Divide $\frac{4}{7}$ by 6; $\frac{5}{12}$ by $\frac{7}{24}$; 16 by $\frac{6}{7}$; and $\frac{5}{9}$ by $\frac{8}{12}$.

A *multiple* of any number is any number which contains it exactly : thus, 15 is a multiple of 3, 24 is a multiple of 8.

The *least common multiple* of several numbers, is the least number which they all will divide without remainder. Thus, the least common multiple of 2, 3, 4 and 6, is 12.

RULE.

To find the least common multiple.

Cancel those numbers which are factors of any of the others.

Divide as many as you can of the remaining numbers by one of the prime numbers, 2, 3, 5, 7, &c.

Proceed in a similar manner with the still remaining numbers, until no number will divide any two.

Multiply all the divisors and all the remaining prime numbers together.

Ex. 1. Find the least common multiple of 3, 8, 9, 6, 4 and 28.

2) 3, 8, 9, 6, 4, 28
2) 4, 9, 3, 14
2, 9, 7

$2 \times 2 \times 2 \times 9 \times 7 = 504$ L. C. M.

Ex. 2. What is the least number which can be divided by 8, 14, 9, 36, 49, 98 and 7?

2) 8, 14, 9, 36, 49, 98, 7
2) 4, 18, 49
2, 9, 49

$2 \times 2 \times 2 \times 9 \times 49 = 3528$ L. C. M.

The principle of this rule is, that the least common multiple of several numbers is produced by multiplying together their simple factors, omitting one of them so long as it is contained in any two of the numbers.

Examples.

Find the least common multiple of the following numbers:

1. 3, 4, 6, 8 and 12. Ans. 24.
2. 4, 18, 12 and 8. Ans. 72.

3. 2, 5, 4, 3, 9 and 25. Ans. 900.
4. 12, 15, 20, 84 and 105. Ans. 420.
5. 36, 63, 28 and 84. Ans. 252.
6. 1, 2, 3, 4, 5, 6, 7, 8 and 9. Ans. 2520.
7. 4, 9, 10, 12 and 18. Ans. 180.
8. 6. 8, 11, 16 and 20. Ans. 2640.
9. 144, 360 and 864. Ans. 4320.
10. 10, 16, 14, 18 and 11. Ans. 55440.
11. 14, 21, 28 and 35.
12. 7, 9, 12 and 15.
13. 24, 39, 104 and 376.
14. 45, 36, 84, 76, 24 and 98.

A *measure* of a number is any divisor which divides it without remainder: thus, 4 is a measure of 12.

The *greatest common measure* of any numbers is the greatest that divides them all without remainder: thus, 6 is the greatest common measure of 12 and 18; 4 is the greatest common measure of 12, 16, and 32.

RULE.

To find the greatest common measure of two numbers.

Divide the greater by the less, and that divisor by the remainder, and so on till nothing remains. The last divisor is the greatest common measure.

Ex. 1. Find the greatest common measure of 45 and 240.

```
45) 240 (5
    225
     15) 45 (3
         45
         --
```

Ans. 15 is the G. C. M.

This rule depends upon the fact that, if a number measures two others, it will also measure their sum, difference, and any multiple of each.

Ex. 2. Find the greatest common measure of 176 and 48.

```
48) 176 (3
    144
    ---
     32) 48 (1
         32
         --
         16) 32 (2
             32
             --
```

Ans. 16 = G. C. M.

If there be more than two numbers, find the greatest common measure of two of them as above, and then that of that common measure and the third number, and so on.

Ex. 3. Find the greatest common measure of 176, 48, and 42. As above, the G. C. M. of 176 and 48 is 16.

```
16) 42 (2
    32
    --
10) 16 (1
    10
    --
 6) 10 (1
     6
     --
 4)  6 (1
     4
     --
 2)  4 (2
     4
     --
```

Ans. 2 is the G. C. M. of 176, 48, and 42.

Examples.

Find the greatest common measure of the following numbers:

1.	126 and 144.	Ans. 18.
2.	3444 and 3556.	Ans. 28.
3.	14186 and 13667.	Ans. 173.
4.	43365 and 44688.	Ans. 147.
5.	11050 and 35581.	
6.	16, 24, and 140.	

RULE.

To reduce a fraction to its lowest terms.

Divide both numerator and denominator by their greatest common measure. Or divide them by any numbers in succession, which will divide them both without remainder.

Ex. 1. Reduce $\frac{5184}{6912}$ to its lowest terms.

$$\begin{array}{r} 5184)\ 6912\ (1 \\ \underline{5184} \\ 1728)\ 5184\ (3 \\ \underline{5184} \end{array}$$

The G. C. M. is 1728; and, dividing both terms of the fraction by it, we have

$$\frac{5184}{6912} = \frac{3}{4} \text{ Ans.}$$

Or $\frac{5184}{6912} = \frac{432}{576} = \frac{36}{48} = \frac{3}{4}$ Ans., where we divide successively by 12, 12, and 12.

Ex. 2. Reduce $\frac{825}{960}$ to its lowest terms.

5 will evidently divide both terms.

Therefore $\frac{825}{960} = \frac{165}{192} = \frac{55}{64}$ Ans., where we divide successively by 5 and 3.

This rule depends upon the principle that the value of a fraction is not changed by dividing both its terms by the same number.

Examples.

Reduce the following fractions to their lowest terms:

1. $\frac{4}{8}, \frac{14}{35}, \frac{48}{60}, \frac{207}{246}$ and $\frac{4494}{8904}$. Ans. $\frac{1}{2}, \frac{2}{5}, \frac{4}{5}, \frac{69}{82}$ and $\frac{107}{212}$.

2. $\frac{24}{30}, \frac{28}{70}, \frac{435}{957}, \frac{4968}{5904}$ and $\frac{26664}{1210788}$

Ans. $\frac{4}{5}, \frac{2}{5}, \frac{5}{11}, \frac{69}{82}$ and $\frac{22}{999}$.

3. $\frac{21}{28}, \frac{56}{63}, \frac{2560}{5760}, \frac{5665}{5720}$, and $\frac{19557}{156933}$.

4. $\frac{621}{738}, \frac{320}{720}, \frac{3444}{3556}, \frac{22288}{40320}$, and $\frac{43365}{44688}$.

RULE.

To reduce an improper fraction to an equivalent whole or mixed number.

Divide the numerator by the denominator; and write the remainder, if any, over the denominator.

Ex. 1. Reduce $\frac{2504}{9}$ to a whole or mixed number.

9) 2504

278⅔

$\therefore \frac{2504}{9} = 278\frac{2}{9}$ Ans. *

Ex. 2. Reduce $\frac{5184}{144}$ to an equivalent whole or mixed number.

144) 5184 (36
432

864
864

$\therefore \frac{5184}{144} = 36$ Ans.

Ex. 3. Reduce $\frac{116}{16}$ to a whole or mixed number.

16) 116 (7
112

4

$\therefore \frac{116}{16} = 7\frac{4}{16} = 7\frac{1}{4}$ Ans.

This rule depends upon the fact that the denominator of a fraction is the divisor of the numerator.

Examples.

Reduce the following improper fractions to whole or mixed numbers equivalent to them:

1. $\frac{213}{17}$, $\frac{1245}{22}$, and $\frac{4076361}{2019}$. Ans. $12\frac{9}{17}$, $56\frac{13}{22}$, and 2019.

2. $\frac{375}{13}$, $\frac{203}{8}$, $\frac{3797}{29}$, and $\frac{9960}{176}$.

Ans. $28\frac{11}{13}$, $25\frac{3}{8}$, $130\frac{27}{29}$, and $56\frac{13}{22}$.

* The sign ∴ is used instead of the word *therefore*.

3. $\frac{157}{10}$, $\frac{59}{4}$, $\frac{729}{15}$, and $\frac{40193}{241}$.

4. $\frac{29}{3}$, $\frac{354}{24}$, $\frac{11500}{397}$, and $\frac{728164}{1283}$.

RULE.

To reduce a mixed number to a simple fraction.

Multiply the whole number by the denominator of the fraction, add in the numerator, and place the sum over the denominator.

Ex. 1. Reduce $25\frac{4}{7}$ to a simple fraction.

$$\begin{array}{r} 25 \\ 7 \\ \hline 175 \\ 4 \\ \hline 179 \end{array}$$

$\therefore 25\frac{4}{7} = \frac{179}{7}$. Ans.

Ex. 2. Reduce $143\frac{3}{29}$ to a simple fraction.

$$\begin{array}{r} 143 \\ 29 \\ \hline 1287 \\ 286 \\ \hline 4147 \\ 3 \\ \hline 4150 \end{array}$$

$\therefore 143\frac{3}{29} = \frac{4150}{29}$. Ans.

This rule is the converse of the preceding one.

Examples.

Reduce the following mixed numbers to simple fractions:

1. $16\frac{6}{7}$, $54\frac{5}{12}$, $514\frac{5}{16}$, and $976\frac{1}{10}$.

Ans. $\frac{118}{7}$, $\frac{653}{12}$, $\frac{8229}{16}$, and $\frac{9767}{10}$

2. $23\frac{11}{12}$, $132\frac{7}{16}$, $100\frac{19}{59}$, and $47\frac{3147}{8400}$.

Ans. $\frac{287}{12}$, $\frac{2119}{16}$, $\frac{5919}{59}$, and $\frac{397947}{8400}$.

3. $4\frac{1}{2}$, $2\frac{3}{10}$, $25\frac{7}{18}$, and $402\frac{21}{700}$.

4. $111\frac{11}{111}$, $202\frac{107}{2020}$, $5\frac{3}{1000}$, and $17\frac{26}{941}$.

RULE.

To reduce a complex fraction to an equivalent simple one.

Reduce, if necessary, each of its terms to a simple fraction; then multiply the extremes for a numerator, and the means for a denominator.

Ex. 1. Reduce $\frac{3\frac{2}{5}}{10\frac{4}{11}}$ **to a simple fraction.**

the numerator $3\frac{2}{5} = \frac{17}{5}$, the denominator $10\frac{4}{11} = \frac{114}{11}$

$\therefore$ the fraction $= \frac{\frac{17}{5}}{\frac{114}{11}}$

$17 \times 11 = 187 =$ product of extremes, the new numerator,

$114 \times 5 = 570 =$ product of means, the new denominator;

$$\therefore \frac{3\frac{2}{5}}{10\frac{4}{11}} = \frac{\frac{17}{5}}{\frac{114}{11}} = \frac{187}{570}. \quad \text{Ans.}$$

Ex. 2. Reduce $\frac{2\frac{1}{4}}{5\frac{2}{3}}$, $\frac{5\frac{1}{2}}{\frac{3}{4}}$, $\frac{\frac{2}{7}}{\frac{3}{5}}$, and $\frac{12}{2\frac{1}{3}}$ severally to simple fractions.

$$\frac{2\frac{1}{4}}{5\frac{2}{3}} = \frac{\frac{9}{4}}{\frac{17}{3}} = \frac{27}{68}. \quad \text{Ans.}$$

$$\frac{5\frac{1}{2}}{\frac{3}{4}} = \frac{\frac{11}{2}}{\frac{3}{4}} = \frac{44}{6} = \frac{22}{3}. \quad \text{Ans.}$$

$$\frac{\frac{2}{7}}{\frac{3}{5}} = \frac{10}{21}. \quad \text{Ans.}$$

$$\frac{12}{2\frac{1}{3}} = \frac{\frac{12}{1}}{\frac{7}{3}} = \frac{36}{7}. \quad \text{Ans.}$$

This rule depends upon the fact that the denominator of a fraction is the division of the numerator.

Thus, $\frac{12}{2\frac{1}{3}} = 12 \div 2\frac{1}{3} = 12 \div \frac{7}{3} = 12 \times \frac{3}{7} = \frac{36}{7}$.

This example, and any similar one, may be conveniently reduced by multiplying both its terms by the denominator of the fractional part of the mixed number.

Thus, $\frac{12}{2\frac{1}{3}} = \frac{36}{7}$, for $12 \times 3 = 36$, and $2\frac{1}{3} \times 3 = 7$.

$$2\frac{1}{3} \times 3 = 6 + \frac{3}{3} = 6 + 1 = 7.$$

Similarly, $\frac{4\frac{2}{5}}{7} = \frac{22}{35}$.

Examples.

Reduce the following complex fractions to simple ones.

1. $\frac{2\frac{1}{2}}{4\frac{2}{5}}$, $\frac{3\frac{2}{3}}{1\frac{3}{4}}$, $\frac{\frac{3}{5}}{10}$ and $\frac{1\frac{4}{7}}{\frac{4}{9}}$ Ans. $\frac{35}{66}$, $\frac{44}{21}$, $\frac{3}{50}$ and $\frac{99}{28}$.
2. $\frac{47\frac{5}{8}}{94}$, $\frac{137}{11\frac{4}{11}}$, $\frac{\frac{3}{7}}{\frac{5}{9}}$ and $\frac{\frac{4}{3}}{10\frac{1}{9}}$ Ans. $\frac{381}{752}$, $\frac{1507}{4}$, $\frac{27}{35}$ and $\frac{12}{91}$.
3. $\frac{4\frac{1}{5}}{7\frac{1}{8}}$, $\frac{500}{6\frac{4}{11}}$, $\frac{25\frac{3}{4}}{\frac{1}{8}}$ and $\frac{\frac{7}{10}}{1\frac{1}{2}}$.
4. $\frac{\frac{4}{9}}{\frac{3}{16}}$, $\frac{2\frac{1}{4}\frac{1}{3}}{200}$, $\frac{20\frac{1}{20}}{20}$ and $\frac{408}{1\frac{2}{10}\frac{3}{0}}$.

RULE.

To reduce a compound fraction to a single one.

Multiply the numerators together for a numerator, and the denominators for a denominator. Cancel, if possible, before multiplying.

Ex. 1. Reduce $\frac{2}{3}$ of $\frac{7}{9}$, $\frac{4}{9}$ of $\frac{12}{7}$, $\frac{8}{33}$ of $1\frac{10}{12}$, and $\frac{3}{5}$ of $\frac{4}{9}$ of $\frac{25}{18}$ of 6 severally to single fractions.

$$\frac{2}{3} \times \frac{7}{9} = \frac{14}{27}. \text{ Ans.}$$

$$\frac{\overset{4}{\cancel{4}}}{\underset{3}{\cancel{9}}} \times \frac{\cancel{12}}{7} = \frac{16}{21}. \text{ Ans.}$$

$$\frac{8}{33} \times 1\frac{10}{12} = \frac{\overset{2}{\cancel{8}}}{\underset{3}{\cancel{33}}} \times \frac{\overset{2}{\cancel{22}}}{\underset{3}{\cancel{12}}} = \frac{4}{9}. \text{ Ans.}$$

$$\frac{\cancel{3}}{\cancel{5}} \times \frac{4}{9} \times \frac{\overset{5}{\cancel{25}}}{\underset{3}{\cancel{18}}} \times \frac{\cancel{6}}{1} = \frac{20}{9}. \text{ Ans.}$$

This rule depends upon the fact that a compound fraction is the *product* of two or more single ones.

Examples.

Reduce the following compound fractions to equivalent single ones.

1. $\frac{2}{3}$ of $\frac{5}{7}$, $\frac{3}{4}$ of $\frac{2}{3}$, $\frac{1}{2}$ of $6\frac{2}{5}$, and $\frac{1}{3}$ of $\frac{3}{5}$ of 20.

Ans. $\frac{10}{21}$, $\frac{1}{2}$, $\frac{16}{5}$ and 4.

2. $\frac{7}{8}$ of $1\frac{1}{7}$, $\frac{3}{7}$ of $\frac{5}{12}$ of $\frac{1}{8}$, $\frac{2}{3}$ of $\frac{3}{5}$ of $12\frac{1}{2}$, and $\frac{3}{5}$ of £2 10*s.*

Ans. 1, $\frac{5}{224}$, 5, and £$\frac{3}{4}$.

3. $\frac{4}{5}$ of $\frac{2\frac{1}{4}}{12}$, $\frac{3}{7}$ of $8\frac{2}{5}$, $\frac{2}{33}$ of $6\frac{2}{7}$ of 11, and $\frac{2}{3}$ of $\frac{4}{5}$ of $\frac{3}{4}$.

4. $\frac{7}{18}$ of $\frac{4}{35}$ of $12\frac{1}{2}$, $\frac{3}{4}$ of $\frac{5}{11\frac{1}{4}}$ of 2, $\frac{6}{17}$ of $5\frac{1}{3}$, and $\frac{60}{751}$ of $\frac{3\frac{2}{3}}{4\frac{3}{9}}$.

RULE.

To reduce fractions to others of the same value having one common denominator.

Find the least common multiple of the denominators, and then multiply the terms of each fraction by such a number as will make the denominator the same as the least common multiple.

The multiplier is readily found by dividing the least common multiple by the denominator.

Ex. 1. Reduce $\frac{4}{9}$, $\frac{7}{12}$ and $\frac{5}{8}$ to equivalent fractions having one common denominator.

3) 9, 12, 8

3, 4, 8 $3 \times 3 \times 8 = 72$ L. C. M.

$\frac{4}{9} \times \frac{8}{8} = \frac{32}{72}$, 9's in 72, 8 times; multiply $\frac{4}{9}$ by $\frac{8}{8}$

$\frac{7}{12} \times \frac{6}{6} = \frac{42}{72}$, 12's in 72, 6 times; multiply $\frac{7}{12}$ by $\frac{6}{6}$

$\frac{5}{8} \times \frac{9}{9} = \frac{45}{72}$, 8's in 72, 9 times; multiply $\frac{5}{8}$ by $\frac{9}{9}$

$\therefore \frac{32}{72}, \frac{42}{72}, \frac{45}{72}$. Ans.

Ex. 2. Reduce 6, $2\frac{1}{5}$, $\frac{5}{12}$, and $\frac{1}{3}$ of $\frac{4}{5}$ to fractions of the same value with one common denominator.

$\frac{6}{1}, \frac{11}{5}, \frac{5}{12}$, and $\frac{4}{15}$

3) 1, 5, 12, 15

4, 5 60 L. C. M.

$$\left.\begin{aligned} \frac{6}{1} \times \frac{60}{60} &= \frac{360}{60} \\ \frac{11}{5} \times \frac{12}{12} &= \frac{132}{60} \\ \frac{5}{12} \times \frac{5}{5} &= \frac{25}{60} \\ \frac{4}{15} \times \frac{4}{4} &= \frac{16}{60} \end{aligned}\right\} \text{ Ans.}$$

Ex. 3. Reduce $1\frac{3}{4}$, $\frac{\frac{2}{3}}{1\frac{1}{2}}$, 10, and $2\frac{1}{5}$ to other fractions of equal value, all having the same denominator.

$\frac{7}{4}, \frac{4}{9}, \frac{10}{1}, \frac{13}{5}$ $\quad 4 \times 9 \times 5 = 180$. L. C. M.

$$\left.\begin{aligned} \frac{7}{4} \times \frac{45}{45} &= \frac{315}{180} \\ \frac{4}{9} \times \frac{20}{20} &= \frac{80}{180} \\ \frac{10}{1} \times \frac{180}{180} &= \frac{1800}{180} \\ \frac{13}{5} \times \frac{36}{36} &= \frac{468}{180} \end{aligned}\right\} \text{ Ans.}$$

The following rule would apply to examples similar to this, viz.: multiply each numerator by all the other denominators for a numerator, and all the denominators together for a denominator.

These rules depend upon the fact that the value of a fraction is not changed by multiplying both its terms by the same number.

Fractions must have a common denominator before we can add, subtract, or compare them.

Examples.

Reduce the following fractions to others equal to them, and having a common denominator.

1. $\frac{2}{3}, \frac{3}{4}$ and $\frac{5}{12}$. Ans. $\frac{8}{12}, \frac{9}{12}$ and $\frac{5}{12}$.

2. $\frac{3}{8}, \frac{4}{5}$ and $\frac{3}{12}$. Ans. $\frac{45}{120}, \frac{96}{120}$ and $\frac{30}{120}$.

3. $\frac{5}{7}$, $\frac{8}{9}$, $\frac{7}{12}$ and $\frac{13}{15}$. Ans. $\frac{900}{1260}$, $\frac{1120}{1260}$, $\frac{735}{1260}$ and $\frac{1092}{1260}$.

4. $\frac{4}{5}$, $2\frac{5}{9}$ and $3\frac{2}{11}$. Ans. $\frac{396}{495}$, $\frac{1265}{495}$ and $\frac{1575}{495}$.

5. $\frac{3}{7}$, $\frac{9}{14}$, $\frac{17}{21}$ and $\frac{25}{28}$ Ans. $\frac{36}{84}$, $\frac{54}{84}$, $\frac{68}{84}$ and $\frac{75}{84}$.

6. $1\frac{1}{2}$, 7, $4\frac{1}{4}$ and $6\frac{1}{6}$. Ans. $\frac{18}{12}$, $\frac{84}{12}$, $\frac{51}{12}$ and $\frac{74}{12}$.

7. $\frac{2\frac{1}{3}}{7}$, $8\frac{1}{4}$, $\frac{9+\frac{1}{11}}{9\times\frac{1}{11}}$ and $16\frac{7}{18}$. Ans. $\frac{12}{36}$, $\frac{297}{36}$, $\frac{400}{36}$ and $\frac{610}{36}$.

8. $\frac{3}{7}$ of $\frac{2}{3}$ of $\frac{5}{8}$ and $\frac{3}{4}$ of $\frac{5}{7}$ of $\frac{3}{5}$.

9. $\frac{4}{7}$ of $2\frac{1}{4}$, 11, $\frac{\frac{3}{4}}{1\frac{1}{4}}$ and $6\frac{4}{7}$.

10. $\frac{4\times\frac{2}{3}}{3+\frac{2}{3}}$, $\frac{1}{2}$ of $\frac{2}{3}$ of 6, 5 and $\frac{3}{4}$ of 2.

11. $\frac{3}{20}$, $\frac{7}{25}$, $\frac{9}{50}$ and $\frac{27}{100}$.

12. $\frac{2-\frac{4}{5}}{2+\frac{4}{5}}$, $\frac{3+\frac{1}{4}}{3-\frac{1}{4}}$, $\frac{5-\frac{1}{10}}{5-\frac{3}{10}}$ and $\frac{\frac{2}{3}+\frac{1}{3}}{\frac{2}{3}-\frac{1}{9}}$.

RULE.

To reduce a fraction or quantity from one name to another without changing its value.

Multiply or divide as in Reduction.

Ex. 1. Reduce $\frac{5}{28}$ £ to the fraction of a penny.

Here we must multiply.

$$£\frac{5}{\cancel{28}_{7}} \times \frac{\overset{5}{\cancel{20}}}{1} \times \frac{12}{1} = \frac{300}{7}d. \quad \text{Ans.}$$

Ex. 2. Reduce $\frac{3}{7}$ inch to the fraction of a yard.

Here we must divide.

$$\text{In. } \frac{\cancel{3}}{7} \times \frac{1}{12} \times \frac{1}{\cancel{3}} = \frac{1}{84} \text{ yard. Ans.}$$

Ex. 3. Express $\frac{6}{11}$ Eng. ells by the fraction of a yard.

Here we must multiply and divide,

$$\text{Eng. ell } \frac{\overset{3}{\cancel{6}}}{11} \times \frac{5}{1} \times \frac{1}{\underset{2}{\cancel{4}}} = \frac{15}{22} \text{ yard. Ans.}$$

Ex. 4. Express $4\frac{1}{2}d.$ by a fraction of a half-crown.

$4\frac{1}{2}d. = \frac{9}{2}d.$, and 1 half-crown $= 30d.$

$$\therefore \frac{\overset{3}{\cancel{9}}}{2} \times \frac{1}{\underset{10}{\cancel{30}}} = \frac{3}{20} \text{ halfcr. Ans.} \qquad \text{Or } \frac{4\frac{1}{2}}{30} = \frac{9}{60} = \frac{3}{20}.$$

Ex. 5. Reduce 9 oz. $2\frac{2}{7}$ dr. to the fraction of a lb. avoirdupois.

oz.	dr.
9	$2\frac{2}{7}$
16	
146	
7	

1024 sevenths of a dram.

$$\frac{\overset{4}{\cancel{1024}}}{7 \times \underset{2}{\cancel{16}} \times \underset{2}{\cancel{16}}} = \frac{4}{7} \text{ lb. Ans.}$$

Examples.

1. Reduce $\frac{1}{320}$ £ to the fraction of a penny. Ans. $\frac{3}{4}d.$

2. Reduce $\frac{4}{5}$ dwt. to the fraction of a pound Troy.

Ans. $\frac{1}{300}$ lb.

3. Reduce $\frac{11}{13}$ gallon of wine to the fraction of a hhd.

Ans. $\frac{11}{819}$ hhd.

4. $\frac{4233600}{71}$ second to the fraction of a week.

Ans. $\frac{7}{71}$ week.

5. What part of a lb. avoirdupois is $\frac{1}{140}$ cwt? Ans. $\frac{4}{5}$ lb.

6. What fraction of a yard is $\frac{2}{3}$ Eng. ell? Ans. $\frac{5}{6}$ yd.

7. Reduce $3\frac{4}{15}$ cr. to the fraction of a guinea. Ans. $\frac{7}{9}$ guin.

8. What part of 5*s*. 9*d*. is 2*s*. 3*d*? Ans. $\frac{9}{23}$.

9. Reduce 4*s*. $6\frac{1}{2}$*d*. to the fraction of a pound. Ans. $\frac{109}{480}$£.

10. Reduce 3 roods 32 perches to the fraction of an acre.

Ans. $\frac{19}{20}$ acre.

11. Reduce $\frac{1}{9}$ shilling to the fraction of 2*d*.

12. Reduce $\frac{2}{3}$ of $7\frac{5}{16}$*d*. to the fraction of a half-crown.

13. What part of 13*s*. 4*d*. is $\frac{3}{4}$ of 6*s*. 8*d*.?

14. Reduce $\frac{5}{6}$ tun to the fraction of a gallon.

15. Reduce 4 bush. $2\frac{2}{7}$ pecks of corn to the fraction of a quarter.

16. Reduce $6\frac{1}{2}$*d*. to the fraction of a half-sovereign.

17. Reduce $\frac{7}{16}$ mile to the fraction of a yard.

18. $5\frac{1}{4}$ square inches to the fraction of a square yard.

19. What fraction of a pipe of wine is 5 gal. $1\frac{1}{2}$ qt.

20 Reduce $\frac{4}{7}$ guinea to the fraction of 4*s.* 6*d.*

RULE.

To find the value of a fraction of given name.

Multiply the numerator by as many of the lower name as make one of the higher, and divide by the denominator.

Ex. 1. Find the value of $\frac{5}{8}$ shilling.

$$\begin{array}{r} 5 \\ 12 \\ \hline 8\,)\,60 \\ \hline 7\ \ 4 \\ \quad 4 \\ \hline 8)16 \\ \hline \quad 2d. \end{array}$$

Ans. $7\frac{1}{2}d.$

Or thus, $\frac{5}{\cancel{8}_{2}} \times \frac{\cancel{12}^{3}}{1} = \frac{15}{2} = 7\frac{1}{2}d.$

Ex. 2. Find the value of $\frac{121}{320}$£.

$$\begin{array}{r} 121 \\ \quad 20 \\ \hline 320\,)\,2420\,(7 \\ \quad 224 \\ \hline \quad 18 \\ \quad 12 \\ \hline 32)216\,(6 \\ \quad 192 \\ \hline \quad 24 \\ \quad\ \ 4 \\ \hline 32)96\,(3 \\ \quad 96 \\ \hline \end{array}$$

Ans. 7*s.* $6\frac{3}{4}d.$

Examples.

1. Find the integral value of $\frac{4}{5}$ shilling, and of $\frac{7}{9}$ cwt.

Ans. $9\frac{1}{2}d.$ $\frac{2}{5}$ f., and 3 qr. $3\frac{1}{9}$ lb.

2 What is the value of $\frac{3}{5}$ lb. Troy, and of $\frac{29}{390}$ crown.

Ans. 7 oz. 4 dwt., and $4\frac{4}{5}$d. $\frac{11}{15}$ f.

3. Find the value of $\frac{7}{40}$ £ and $\frac{5}{8}$ ton.

Ans. 3*s.* 6*d.* and 12 cwt. 2 qr.

4. Find the value of $\frac{15}{32}$ quarter of corn, and $\frac{1}{4}$ of $\frac{2}{7}$ £.

Ans. 3 bush. 3 pk., and 1*s.* $5\frac{1}{7}d.$

5. $\frac{109}{450}$ day, and $\frac{3}{7}$ mile.

Ans. 5 hours 48 m. 48 sec., and 3 fur. 17 po. 2 ft. $4\frac{2}{7}$ in.

6. $\frac{2}{15}$ tun, and $\frac{4}{15}$ of $1\frac{1}{8}$ Eng. ell.

Ans. 33 gal. 2 qt. $0\frac{4}{5}$ pt., and 1 qr. 2 nl.

7. $\frac{2}{15}$ of 4 weeks, and $\frac{4}{3}$ of 2 guineas.

Ans. 3 da. 17 hr. 36 m., and £2 16*s.*

8. $\frac{1}{8}$ of $3\frac{1}{2}$ tuns, and $\frac{3}{7}$ of $1\frac{1}{2}$ cwt.

Ans. 220 gal. 2 qts., and 2 qr. 16 lb.

MISCELLANEOUS.

1. Find the L. C. M. of 3, 4, 12 and 20; also of 2, 9, 12 and 8.

2. Reduce to their simplest form $\frac{18}{42}$, $\frac{720}{488}$, $\frac{48}{560}$, and $\frac{3450}{18450}$.

3. Reduce to the least common denominator $\frac{2}{3}$, $\frac{3}{5}$ and $\frac{7}{11}$; also $\frac{1}{2}$ of $3\frac{1}{2}$, $4\frac{2}{3}$ and $\frac{5}{8}$; also $\frac{1\frac{1}{4}}{6}$, $\frac{7}{12}$, $\frac{\frac{2}{3}}{4\frac{1}{2}}$ and 9; and also $\frac{4}{5}$ of $\frac{1}{3}$, $7\frac{1}{2}$ and $\frac{5}{7}$.

4. Reduce $\frac{7}{180}$£ to the fraction of a penny; $\frac{5}{11}$yd. to the fraction of an inch; $\frac{250}{3}$ farthing to the fraction of a shilling; and $7\frac{1}{2}d.$ to the fraction of 12*s.* 6*d.*

5. Find the integral value of $\frac{5}{16}$£; $\frac{4}{7}$ crown; $\frac{2}{9}$ of $\frac{12}{5}$ of 60 yds. cloth; $\frac{7}{16}$ ton; $\frac{5}{9}$ dwt.; and $\frac{2}{5}$ of $2\frac{3}{4}$ cwt.

6. Reduce $\frac{4}{9}$, $\frac{5}{8}$ and $\frac{11}{12}$ to the least common denominator; $\frac{235}{580}$ to its lowest terms; $\frac{11}{80}$ pipe to the fraction of a gallon; and $\frac{7}{16}$ mile to its integral value.

ADDITION OF FRACTIONS.

RULE.

Reduce them when necessary, to *simple fractions*, to the *same name*, and to the *least common denominator*, add the numerators, and place their sum over the common denominator.

Ex. 1. Add together $\frac{2}{5}$ and $\frac{4}{5}$.

Here no reduction being necessary, $\frac{2}{5}+\frac{4}{5}=\frac{6}{5}=1\frac{1}{5}$ Ans.

Ex. 2. Find the sum of $3\frac{2}{3}$, 5, and $\frac{4}{7}$ of $\frac{1}{3}$.

$$\frac{11}{3}+\frac{5}{1}+\frac{4}{21}=$$

$$\frac{77}{21}+\frac{105}{21}+\frac{4}{21}=\frac{186}{21}=\frac{62}{7}=8\frac{6}{7}\text{ Ans.}$$

Ex. 3. Add together $\frac{2}{3}$£, $\frac{5}{6}s.$ and $\frac{3}{8}d.$

$$\left.\begin{aligned} \tfrac{2}{3}£ &= \tfrac{2}{\cancel{3}} \times \frac{20}{1} \times \frac{\overset{4}{\cancel{12}}}{1} = 160d. \\ \tfrac{5}{6}s. &= \tfrac{5}{\cancel{6}} \times \frac{\overset{2}{\cancel{12}}}{1} \quad = 10d. \\ \tfrac{3}{8}d. &= \qquad\qquad\quad = \tfrac{3}{8}d. \end{aligned}\right\} = 170\tfrac{3}{8}d. = 14s.\ 2\tfrac{3}{8}d. = 14s.\ 2\tfrac{1}{4}d.\ \tfrac{1}{2}f.\ \text{Ans.}$$

Examples of this kind may also be worked thus:

£	s.	d.
2	5	3
20	12	4
3) 40	6) 60	8) 12
13 – 1	10	$1\frac{4}{8} = 1\frac{1}{2}$
12		
3) 12		
4		

$\therefore \frac{2}{3}£ = 13s.\ 4d.$, $\therefore \frac{5}{6}s. = 10d.$ $\therefore \frac{3}{8}d. = 1\frac{1}{2}f.$

$\frac{5}{6}s. = \quad 10d.$

$\frac{3}{8}d. = \quad 1\frac{1}{4}d.\ \frac{1}{2}f.$

$14s.\ 2\frac{1}{4}d.\ \frac{1}{2}f.$ Ans.

Ex. 4. Find the sum of $\frac{\frac{5}{7}}{1\frac{1}{4}}$ cwt.; $\frac{2}{5}$ of $1\frac{5}{8}$ qr.; and $7\frac{1}{7}$ lb.

$$\frac{\frac{5}{7}}{1\frac{1}{4}}\text{ cwt.} = \frac{\frac{5}{7}}{\frac{5}{4}} = \frac{20}{35} = \frac{4}{7}\text{ cwt.} = \frac{4}{\cancel{7}} \times \frac{\overset{16}{\cancel{112}}}{1} = 64\text{ lb.}$$

$\frac{2}{5}$ of $1\frac{5}{6}$ qr. $= \frac{2}{5} \times \frac{11}{\cancel{6}_{3}} = \frac{11}{15}$ qr. $= \frac{11}{15} \times \frac{28}{1} = \frac{308}{15}$ lb. $= 20\frac{8}{15}$ lb.

$7\frac{1}{7}$ lb. $= 7\frac{1}{7}$ lb.

$\therefore 64 + 20\frac{8}{15} + 7\frac{1}{7} = 91^{*} + \frac{8}{15} + \frac{1}{7} = 91 + \frac{56}{105} + \frac{15}{105} =$

$91\frac{71}{105}$ lb. $= 3$ qrs. $7\frac{71}{105}$ lb. Ans.

Ex. 5. Add together $\frac{3}{16}$, $\frac{5}{12}$, $\frac{4}{15}$ and $\frac{19}{40}$.

2) 16, 12, 15, 40

2) 8, 6, 15, 20 $\therefore 2 \times 2 \times 5 \times 4 \times 3 = 240$ L. C. M.

5) 4, 3, 15, 10

4, 3, 3

$240 \div 16 = 15 \quad \therefore \frac{3}{16} \times \frac{15}{15} = \frac{45}{240}$

$240 \div 12 = 20 \quad \therefore \frac{5}{12} \times \frac{20}{20} = \frac{100}{240}$

$240 \div 15 = 16 \quad \therefore \frac{4}{15} \times \frac{16}{16} = \frac{64}{240}$

$240 \div 40 = 6 \quad \therefore \frac{19}{40} \times \frac{6}{6} = \frac{114}{240}$

$= \frac{323}{240} = 1\frac{83}{240}$ Ans.

The reason of this rule is obvious from what has been said before.

Examples.

Find the sum of the following fractions:

1. $\frac{3}{10} + \frac{9}{10}$. Ans. $1\frac{1}{5}$.

2. $12\frac{4}{5} + 5\frac{1}{6}$. Ans. $17\frac{29}{30}$.

3. $\frac{3}{4} + \frac{2}{7} + \frac{2}{5}$. Ans. $1\frac{61}{140}$.

* It is frequently convenient to take the *sum* of the integral parts, and annex that of the fractional parts, as in this instance.

4. $2\frac{1}{2} + \frac{3}{4}$ of $5\frac{1}{2} + 4\frac{7}{11} + 5$. Ans. $16\frac{83}{88}$.

5. $4\frac{7}{8} + \frac{5}{6} + \frac{1}{2}$ of $1\frac{1}{4}$. Ans. $6\frac{1}{3}$.

6. $\frac{2}{5} + \frac{5}{8} + \frac{3}{7}$. Ans. $1\frac{227}{280}$.

7. $3\frac{5}{7} + \frac{5}{11} + 4\frac{5}{8}$. Ans. $8\frac{489}{616}$.

8. $\frac{3}{4}$ of $19 + \frac{1}{9}$ of $\frac{3}{5} + \frac{5}{8}$ of 12. Ans. $21\frac{49}{60}$.

9. $\frac{4\frac{1}{3}}{7} + \frac{2}{1\frac{5}{9}} + \frac{\frac{1}{6}}{\frac{7}{11}}$. Ans. $2\frac{1}{6}$.

10. $\frac{5}{9}£ + \frac{3}{8}s. + \frac{5}{7}d.$ Ans. $11s.\ 6\frac{1}{2}d.\ \frac{4}{21}f.$

11. $\frac{1}{5}$ lb. troy $+ \frac{1}{8}$ oz. Ans. 2 oz. 10 dwt. 12 gr.

12. $\frac{5}{6}$ of 3 Eng. ells $+ \frac{1}{3}$ of $1\frac{1}{4}$ yard.
Ans. 2 Eng. ells 4 qrs. $0\frac{2}{3}$ nls.

13. $\frac{4}{5} + \frac{7}{10} + \frac{8}{15}$; and $\frac{1}{6} + \frac{3}{8} + \frac{7}{10}$.

14. $5\frac{1}{3} + 2\frac{2}{7} + 7$; and $\frac{2}{3}$ of $\frac{1}{3} + \frac{3}{5}$ of $\frac{4}{9}$.

15. $\frac{3}{7}$ of $3\frac{2}{3} + \frac{1}{2}$ of $6\frac{3}{7}$; and $\frac{7}{11} + 2\frac{3}{8} + \frac{1}{3}$ of $2\frac{5}{6} + 4$.

16. $\frac{5}{2\frac{1}{2}} + \frac{3\frac{1}{2}}{7} + \frac{2\frac{1}{3}}{21}$; and $\frac{11}{24} + \frac{15}{28} + 2\frac{3}{8} + 40$.

17. $\frac{1}{5}$ hour $+ \frac{1}{4}$ wk. $+ \frac{1}{3}$ day; and $\frac{5}{11}$ mile $+ \frac{2}{5}$ yard $+ \frac{3}{7}$ foot.

18. $\frac{5}{12}$ cwt. $+ \frac{4}{9}$ ton; and $\frac{7}{10}s. + \frac{2}{9}d. + \frac{1}{3}£$.

19. $\frac{2}{3}$ of £2 5*s.* + $\frac{1}{3}$ of $\frac{1}{2}$ of £20 8*s.*; and $\frac{2}{3}$ of £17 17*s.* 6*d.* + $\frac{4}{5}$ of $1\frac{1}{3}$ £ + $\frac{5}{8}$ crown.

20. $\frac{4}{5}$ of 3 a. 1 r. 20 p. + $\frac{3}{8}$ acre + $\frac{3}{4}$ of 3 r. 15 p.; and $\frac{2}{7}$ hhd. + $\frac{3}{4}$ tun + $\frac{5}{6}$ pipe.

SUBTRACTION OF FRACTIONS.

RULE.

Reduce them as in Addition, subtract the less numerator from the greater, and place the difference over the common denominator.

Ex. 1. From $\frac{8}{11}$ take $\frac{3}{11}$. $\frac{8}{11} - \frac{3}{11} = \frac{5}{11}$ Ans.

Ex. 2. From $3\frac{4}{5}$ take $\frac{3}{4}$ of $2\frac{1}{3}$.

$$3\tfrac{4}{5} - \frac{3}{4} \text{ of } 2\tfrac{1}{3} = \frac{19}{5} - \frac{3}{4} \text{ of } \frac{7}{3} = \frac{19}{5} - \frac{7}{4} =$$

$$\frac{76}{20} - \frac{35}{20} = \frac{41}{20} = 2\tfrac{1}{20} \text{ Ans.}$$

Ex. 3. Subtract $\frac{3}{8}$*s.* from $\frac{3}{8}$£.

$$\frac{3}{8}£ = \frac{3}{\cancel{8}_{2}} \times \frac{\overset{5}{\cancel{20}}}{1} \times \frac{\overset{6}{\cancel{12}}}{1} = 90d. = 7s.\ 6d.$$

$$\frac{3}{8}s. = \frac{3}{\cancel{8}_{2}} \times \frac{\overset{3}{\cancel{12}}}{1} = \frac{9}{2}d. = \quad 4\tfrac{1}{2}d.$$

diff. = 7*s.* $1\frac{1}{2}$*d.* Ans.

Or thus,

£	s.
3	3
20	12
8) 60	8) 36
7 - 4	$4\frac{1}{2} = 4\frac{1}{2}d.$

12

8) 48

6

$\therefore \frac{3}{8}£ = 7s.\ 6d.$

$\frac{3}{8}s. = 4\frac{1}{2}d.$

$7s.\ 1\frac{1}{2}d.$ Ans.

The reason of this rule is obvious from what has been said before.

Examples.

Find the difference of the following fractions:

1. $\frac{7}{12} - \frac{3}{12}$; and $1\frac{5}{6} - \frac{2}{3}$ of $\frac{5}{8}$. Ans. $\frac{1}{3}$ and $1\frac{5}{12}$.

2. $\frac{3}{5} - \frac{9}{16}$; and $3\frac{5}{8} - \frac{2}{3}$ of $\frac{1\frac{1}{4}}{2}$. Ans. $\frac{3}{80}$ and $3\frac{5}{24}$.

3. $1 - \frac{1}{4}$ of $1\frac{1}{2}$; and $360 - \frac{11}{15}$. Ans. $\frac{5}{8}$ and $359\frac{4}{15}$.

4. $\frac{4}{5}$ of $72 - \frac{2}{3}$ of 15; and $17\frac{3}{5} - 7\frac{2}{5}$. Ans. $47\frac{3}{5}$ and $9\frac{14}{15}$.

5. Find the value of $\frac{1}{2} - \frac{5}{6} + \frac{3}{4} - \frac{13}{15} + \frac{7}{10} + \frac{11}{20}$. Ans. $\frac{4}{5}$.

6. $\frac{3}{2} - \frac{16}{81} - \frac{4}{9} + \frac{2}{3} - 1 + \frac{8}{27}$. Ans. $\frac{133}{162}$.

7. Required the difference of $\frac{3}{5}$ of $\frac{4\frac{1}{6}}{5\frac{1}{7}}$ and $\frac{2}{3}$ of $7\frac{1}{2}$; and of $\frac{13}{5}$ of $\frac{5\frac{1}{2}}{4\frac{1}{3}}$ and $\frac{7\frac{1}{3}}{11}$ of $15\frac{3}{8}$. Ans. $4\frac{37}{72}$ and 7.

8. $\frac{2}{3}$ of 5 guineas $-\frac{3}{4}$ of $\frac{7}{9}$£. Ans. £2 18*s.* 4*d.*

9. $\frac{7}{9}-\frac{3}{7}$; $\frac{5}{6}-\frac{3}{4}$; and $2\frac{1}{3}-1\frac{2}{3}$.

10. $\frac{3}{4}$ of $4\frac{1}{3}-\frac{1}{8}$ of $\frac{4\frac{1}{3}}{2}$; and $111-\frac{10}{11}$.

11. $\frac{3}{8}$ crown $+\frac{3}{4}$ guinea $+\frac{3}{5}$ of 7*s.* 6*d.* $-\frac{3}{4}$ of 2*d.*

12. $\frac{3}{5}$ league $-\frac{5}{2}$ of a quarter of a mile.

13. $\frac{4}{9}$ bushel $-\frac{3}{8}$ gallon.

14. What number is that to which if you add $\frac{1}{2}$ of $9\frac{2}{3}$ the sum will be 81?

The following method is convenient, when applicable.

From 12 take $4\frac{2}{3}$.

$$\left.\begin{array}{r} 12 \\ 4\frac{2}{3} \\ \hline 7\frac{1}{3} \end{array}\right\}$$ Ans.

Practically thus; $\frac{2}{3}$ from 0 I cannot, borrow 1, that is $\frac{3}{3}$, $\frac{2}{3}$ from $\frac{3}{3}$ there remains $\frac{1}{3}$, set down $\frac{1}{3}$ and carry 1, 1 and 4 are 5, 5 from 12 there remain 7, set down 7.

From $4\frac{1}{5}$ take $1\frac{2}{5}$.

$$\left.\begin{array}{r} 4\frac{1}{5} \\ 1\frac{2}{5} \\ \hline 2\frac{4}{5} \end{array}\right\}$$ Ans.

Practically thus; $\frac{2}{5}$ from $\frac{1}{5}$ I cannot, borrow 1, that is $\frac{5}{5}$, $\frac{2}{5}$ from $\frac{5}{5}$ there remain $\frac{3}{5}$, which added to $\frac{1}{5}$ above $=\frac{4}{5}$, set down $\frac{4}{5}$ and carry 1, 1 and 1 are 2, 2 from 4 there remain 2, set down 2.

MULTIPLICATION OF FRACTIONS.

RULE.

Reduce them to *simple fractions*, then multiply the numerators together for a numerator, and the denominators for a denominator. Cancel, if possible, before multiplying.

Ex. 1. Multiply $\frac{5}{12}$ by $\frac{4}{15}$.

$$\frac{\overset{1}{\cancel{5}}}{\underset{3}{\cancel{12}}} \times \frac{\overset{1}{\cancel{4}}}{\underset{3}{\cancel{15}}} = \frac{1}{9} \quad \text{Ans.}$$

Ex. 2. Multiply $\frac{2}{9}$ of $2\frac{2}{5}$ by $3\frac{3}{4}$.

$$\frac{2}{9} \text{ of } 2\tfrac{2}{5} \times 3\tfrac{3}{4} = \frac{2}{\underset{3}{\cancel{9}}} \times \frac{\overset{3}{\cancel{12}}}{\cancel{5}} \times \frac{\cancel{15}}{\cancel{4}} = 2 \quad \text{Ans.}$$

The principle of this rule has been already explained.

Examples.

Find the product of the following fractions:

1. $\frac{14}{15} \times \frac{9}{10}$; and $\frac{22}{35} \times \frac{25}{33}$. Ans. $\frac{21}{25}$ and $\frac{10}{21}$.
2. $\frac{392}{675} \times \frac{45}{42}$; and $\frac{5}{8} \times 14\frac{2}{7}$. Ans. $\frac{28}{45}$ and $9\frac{1}{36}$.
3. $20\frac{2}{3}\frac{2}{9} \times 4\frac{7}{8}$; $9 \times \frac{5}{6}$ of $\frac{3}{8}$ of $\frac{6}{7}$. Ans. 100 and $2\frac{2}{3}\frac{3}{8}$.
4. $\frac{9}{10}$ of $1\frac{1}{2} \times \frac{20}{27}$ of $\frac{4}{5}$ of $1\frac{1}{4}$; and $\frac{\frac{4}{5}}{12} \times \frac{1}{2}$. Ans. 1 and $\frac{1}{30}$.
5. $\frac{2}{3}$ of $\frac{2}{7}$ of $\frac{3}{5}$ of $4\frac{1}{6} \times 5$; and $\frac{3}{5}$ of $3\frac{4}{8} \times \frac{9}{10}$ of $\frac{3\frac{1}{8}}{11}$.
6. $\frac{2}{3} \times 24$, and $\frac{16}{21} \times \frac{7}{8}$ of $1\frac{1}{7}$.

7. Multiply £2 16*s*. 8*d*. by $\frac{2}{3}$; and 4 ft. 9 in. by $2\frac{3}{4}$.

8. Multiply 3 ft. 4 in. by 2 ft. 9 in., and express the product in square feet.

$$3 \text{ ft. } 4 \text{ in.} = 3\tfrac{4}{12} \text{ ft.} = 3\tfrac{1}{3} = \frac{10}{3}$$

$$2 \text{ ft. } 9 \text{ in.} = 2\tfrac{9}{12} \text{ ft.} = 2\tfrac{3}{4} = \frac{11}{4}$$

$$\frac{\overset{5}{\cancel{10}}}{3} \times \frac{11}{\underset{2}{\cancel{4}}} = \frac{55}{6} = = 9\tfrac{1}{6} \text{ square feet.} \quad \text{Ans.}$$

9. Multiply 4 ft. 3 in. by 3 ft. 8 in., and express the product in square feet.

DIVISION OF FRACTIONS.

RULE.

Invert the divisor, and proceed as in Multiplication.

Ex. 1. Divide $\frac{5}{12}$ by $\frac{4}{15}$.

$$\frac{5}{12} \div \frac{4}{15} = \frac{5}{\underset{4}{\cancel{12}}} \times \frac{\overset{5}{\cancel{15}}}{4} = \frac{25}{16} = 1\tfrac{9}{16} \quad \text{Ans.}$$

The *reciprocal* of any number is unity divided by that number; thus the reciprocal of 8 is $\frac{1}{8}$.

The divisor $\frac{4}{15}$, being inverted, becomes $\frac{15}{4}$, which is its *reciprocal.*

Ex. 2. Divide $\frac{7}{22}$ of $8\frac{1}{4}$ by $\frac{3}{5}$ of $\frac{5}{9}$.

$$\frac{7}{22} \text{ of } 8\tfrac{1}{4} = \frac{7}{\underset{2}{\cancel{22}}} = \frac{\overset{3}{\cancel{33}}}{4} = \frac{21}{8}$$

$$\frac{3}{5} \text{ of } \frac{5}{9} = \frac{\cancel{3}}{\cancel{5}} \times \frac{\cancel{5}}{\underset{3}{\cancel{9}}} = \frac{1}{3}$$

$$\therefore \frac{21}{8} \div \frac{1}{3} = \frac{21}{8} \times \frac{3}{1} = \frac{63}{8} = 7\frac{7}{8} \text{ Ans.}$$

The principle of this rule has been already explained.

Examples.

1. Divide $\frac{5}{16}$ by $\frac{7}{12}$; and $\frac{250}{7}$ by $\frac{300}{11}$. Ans. $\frac{15}{28}$ and $1\frac{1}{4}\frac{3}{2}$.

2. $\frac{20}{27} \div 8$; and $97 \div 5\frac{4}{7}$. Ans. $\frac{5}{54}$ and $17\frac{1}{3}\frac{6}{9}$.

3. $2\frac{1}{30} \div 4\frac{5}{14}$; and $\frac{12}{4\frac{1}{3}} \div \frac{\frac{4}{5}}{\frac{2}{3}}$. Ans. $\frac{7}{15}$ and $2_{1}\frac{4}{3}$.

4. $\frac{2}{10}$ of $\frac{3}{2} \div \frac{2}{3}$ of $\frac{9}{11}$ of $\frac{3}{4}$; and $42 \div 56$. Ans. $1\frac{1}{10}$ and $\frac{3}{4}$.

5. Divide 1*s.* $4\frac{1}{2}$*d.* $\frac{5}{6}$*f.* by $\frac{1}{3}$ of $1\frac{1}{4}$. Ans. $6\frac{3}{4}$*d.* $\frac{4}{7}\frac{1}{2}$*f.*

6. $\frac{14}{17} \div \frac{6}{5}$; $5\frac{4}{5} \div 6$; $18 \div \frac{1}{6}$ and $\frac{2\frac{3}{4}}{\frac{1}{4}} \div \frac{2}{3}$ of $\frac{3}{8}$.

7. What part of 48 is $\frac{7}{12}$?

8. Divide 15 a. 2 r. 10 p. by $\frac{7}{10}$; and find how many times £7 14*s.* 6*d.* is contained in £21 17*s.* 9*d.*

9. Exhibit $\dfrac{\left(2+\frac{1}{5}\right) \div \left(3+\frac{1}{7}\right)}{\left(\frac{1}{2}-\frac{1}{3}\right) \times \left(4-3\frac{1}{7}\right)}$ in its simplest form.

PROPORTION OF FRACTIONS.

RULE.

Proceed as in ordinary Proportion: that is,
Let the third term be like the answer;
Let the first and second be of the same name;
Multiply the reciprocal of the 1st by the 2nd and 3rd.

To determine the middle term, reduce, if necessary, the 1st and 2nd to a common denominator.

Ex. 1. If $\frac{7}{12}$ £ be paid for $\frac{5}{8}$ gallon of wine, what will $\frac{1}{6}$ tun cost?

$$\begin{matrix}\text{gal.} & \text{tun.} & £\\ \frac{5}{8} & : \frac{1}{6} & :: \frac{7}{12}\end{matrix}$$

$$\frac{1}{6}\text{ tun} = \frac{1}{6} \times \frac{252}{1} = 42 \text{ gall.}$$

$$\begin{matrix}\text{gal.} & \text{gal.} & £\\ \frac{5}{8} & : \frac{42}{1} & :: \frac{7}{76}\end{matrix}$$

$$\frac{\overset{2}{\cancel{8}}}{5} \times \frac{\overset{14}{\cancel{42}}}{1} \times \frac{7}{\underset{\cancel{3}}{\cancel{12}}} = \frac{196}{5} = £39\tfrac{1}{5} = £39\ 4s. \text{ Ans.}$$

Ex. 2. If $\frac{3}{40}$ guinea be charged for conveying $2\frac{1}{2}$ tons $2\frac{9}{10}$ miles, what must be paid for 1 cwt. 5 miles?

$$\begin{matrix}\text{tons.} & \text{cwt.} & \text{guinea.}\\ 2\tfrac{1}{2} & : 1 & :: \frac{3}{40}\end{matrix}$$

$$\begin{matrix}\text{m.} & \text{m.}\\ 2\frac{9}{10} & : 5\end{matrix}$$

$$\text{or,} \quad \begin{matrix}\text{tons.} & \text{tons.} & \text{guinea.}\\ \frac{5}{2} & : \frac{1}{20} & :: \frac{3}{40}\end{matrix}$$

$$\begin{matrix}\text{m.} & \text{m.}\\ \frac{29}{10} & : \frac{5}{1}\end{matrix}$$

$$\frac{\cancel{2}}{\cancel{5}} \times \frac{\cancel{10}}{29} \times \frac{1}{\cancel{20}} \times \frac{\cancel{5}}{1} \times \frac{3}{40} = \frac{3}{1160} \text{ guin.} = \frac{3 \times 21 \times \overset{6}{\cancel{12}} \times \cancel{4}}{\underset{\cancel{29\emptyset}}{\cancel{116\emptyset}}.\ 145} f =$$

$$\frac{378}{145} f. = \frac{1}{2} d. \ \frac{88}{145} f. \text{ Ans.}$$

Ex. 3. A post has one-fourth of its length in the mud, one-third in the water, and ten feet above the water; find its whole length.

Let its whole length be represented by unity, that is, by 1.

$$\frac{1}{4}+\frac{1}{3}=\frac{3}{12}+\frac{4}{12}=\frac{7}{12}=\text{the part in mud and water.}$$

$$\therefore 1-\frac{7}{12}=\frac{12}{12}-\frac{7}{12}=\frac{5}{12}=\text{the part above water.}$$

$$\frac{5}{12} : 1 :: 10 \text{ feet}$$

$$\frac{12}{\cancel{5}}\times\frac{\overset{2}{\cancel{10}}}{1}=24 \text{ feet.} \quad \text{Ans.}$$

Ex. 4. A, B, and C can perform a piece of work in 12 hours; A and B can do it in 16 hours, and A and C in 18 hours: what part of the work can B and C do in 9½ hours?

Since A and B can do the whole in 16 hours,

$$\therefore \text{A and B can do } \frac{1}{16} \text{ in 1 hour;}$$

$$\therefore \text{A and B can do } \frac{12}{16} \text{ or } \frac{3}{4} \text{ in 12 hours,}$$

but A, B and C can do the whole, that is, 1 or $\frac{4}{4}$ in 12 hours,

$$\therefore \text{C can do } \frac{4}{4}-\frac{3}{4}=\frac{1}{4} \text{ in 12 hours,}$$

$$\therefore \text{C can do the whole in 48 hours.}$$

Again, A and C can do $\frac{1}{18}$ in 1 hour,

$$\therefore \text{A and C can do } \frac{48}{18} \text{ or } \frac{8}{3} \text{ in 48 hours,}$$

$$\therefore \frac{8}{3}-1=\frac{8}{3}-\frac{3}{3}=\frac{5}{3}=\text{part A can do in 48 hours,}$$

$$\therefore \frac{1}{4} \text{ of } \frac{5}{3}=\frac{5}{12}=\text{part A can do in 12 hours,}$$

and since A, B and C can do the whole in 12 hours,

$$\therefore \text{ B and C can do } 1 - \frac{5}{12} = \frac{12}{12} - \frac{5}{12} = \frac{7}{12} \text{ in 12 hours.}$$

hrs. hrs. work.

$$\text{Now, } 12 : 9\tfrac{1}{7} :: \frac{7}{12}$$

$$\text{or, } \frac{12}{1} : \frac{64}{7} :: \frac{7}{12}$$

$$\frac{1}{\cancel{12}\,3} \times \frac{\overset{4}{\cancel{64}\,16}}{7} \times \frac{7}{\cancel{12}\,3} = \frac{4}{9} \text{ work. Ans.}$$

Examples.

1. If $\frac{9}{10}$ yard cost $\frac{3}{4}$£, how much may be bought for $\frac{5}{8}$£? Ans. $\frac{3}{4}$ yd.

2. If $\frac{3}{8}$ of a ship cost £1840, what must $\frac{5}{32}$ of it be sold for to gain £20? Ans. £1553 6*s.* 8*d.*

3. How many yards of carpet, $2\frac{3}{4}$ feet wide, will cover a floor $18\frac{3}{4}$ feet broad and $23\frac{1}{4}$ feet long? Ans. $183\frac{2}{3}\frac{1}{3}$ yds.

4. How many Spanish dollars, each worth $41\frac{1}{2}$*d.* sterling, are equivalent to £74 9*s.* 6*d.* sterling? Ans. 432 dollars.

5. How much in length, that is $7\frac{3}{16}$ inches broad, will make a square foot? Ans. $20\frac{4}{11\,5}$ inches.

6. If the carriage of 60 cwt., 20 miles, cost £$14\frac{1}{2}$, what weight ought to be carried 30 miles for £$5\frac{7}{16}$? Ans. 15 cwt.

7. If £100 in 12 months gain £$5\frac{1}{4}$ interest, what principal will gain £$3\frac{3}{8}$ interest in 9 months? Ans. £85 14*s.* $3\frac{1}{4}$*d.* $\frac{4}{7}$*f.*

8. If £$13\frac{1}{3}$ gain £$1\frac{1}{3}\frac{1}{3}$ in 9 months, in what time will £$2\frac{31}{144}$ be gained by £50? Ans. 9 months.

9. If 9 students spend £$10\frac{7}{8}$ in 18 days, how much will 20 students spend at the same rate in 30 days?

10. If a penny loaf weigh 7 oz., when a bushel of wheat costs $5\frac{1}{2}$*s.*, what is the bushel worth when a penny loaf weighs $2\frac{1}{2}$ oz.?

11. A can dig a potato field in 2 days, and B in 3; in what time can A and B together dig it?

12. A can do a piece of work in $7\frac{1}{2}$ days, and B in $8\frac{1}{2}$; in what time can both together do it?

13. A tank is filled by 3 pipes, in 4, $8\frac{4}{7}$, and 15 hours respectively; in what time will they all together fill it?

14. If A in 2 days can do as much as C in 3 days, and B in 5 days as much as C in 4 days, what time will B require to execute a piece of work which A can accomplish in 6 weeks?

15. After detaching $\frac{11}{28}$ and afterwards $\frac{2}{7}$ of an army, the general had 1350 men left; what was his original force?

16. A and B can do a piece of work in 18 days, A and C in 12, and B and C in 9; find the time in which A, B and C together can do it.

17. If $\frac{11}{20}$ of an estate be left by a gentleman to his daughter, and the remainder to his son, and if the difference of their fortunes be £5250, find the value of the estate.

18. If a person spend $\frac{5}{9}$ of his income in board and lodging, $\frac{1}{6}$ in clothes, and save £60 a year, what is his annual income?

What is a fraction? Define and explain the following terms: denominator, numerator, proper fraction, improper fraction, mixed number, compound fraction, complex fraction, simple fraction. By what two operations may the multiplication of a fraction by a whole number be performed? By what two operations may the division of a fraction by a whole number be performed? What effect is produced upon the value of a fraction by multiplying or dividing both its terms by the same number? Define and explain the terms measure, common measure, greatest common measure, multiple, common multiple, least common multiple. What kind of fraction is $\frac{1}{8}$ of $\frac{5}{12}$? $4\frac{2}{7}$? $\frac{11}{3}$? $\frac{\frac{4}{5}}{1\frac{1}{2}}$? and $\frac{4}{5}$? What is the G. C. M. of $\frac{16}{20}$, of $\frac{25}{80}$, and of $\frac{19}{76}$? What is the L. C. M. of 2, 5, and 8? of 4, 6, and 10? of 2, 3, and 5? What is the object of find-

ing the G. C. M.? Reduce to their lowest terms $\frac{6}{12}$, $\frac{8}{10}$, $\frac{14}{42}$, $\frac{24}{36}$, $\frac{50}{1000}$, and $\frac{49}{56}$. What is the object of reducing fractions to a common denominator? Reduce $\frac{1}{3}$ and $\frac{1}{2}$ to a common denominator; also $\frac{3}{4}$ and $\frac{2}{3}$, also $\frac{1}{10}$, $\frac{2}{5}$, and $\frac{1}{15}$. What sign may be substituted in a compound fraction for the word *of*? What do you understand by cancelling? Reduce to single fractions $\frac{2}{3}$ of $\frac{3}{4}$, $\frac{1}{5}$ of $\frac{4}{3}$, and $\frac{3}{4}$ of 10. On what principle does the rule for reducing fractions to a common denominator depend? What is the reciprocal of $\frac{4}{5}$? of 10? of $\frac{1}{6}$? and of 6? Which is the greatest $\frac{2}{3}$, $\frac{1}{2}$, or $\frac{3}{5}$? What is the nearest whole number to $4\frac{5}{8}$? to $6\frac{2}{3}$? to $20\frac{7}{16}$? Name several fractions each equal to $\frac{5}{10}$. What part of $\frac{1}{10}$ is $\frac{1}{100}$? What part of $\frac{1}{100}$ is $\frac{1}{1000}$. Reduce to their lowest terms $\frac{5}{10}$, $\frac{25}{100}$ and $\frac{75}{100}$.

DECIMALS.

A decimal is a fraction whose denominator is 10, or 100, or 1000, &c., and whose numerator consists of as many figures as there are ciphers in the denominator.

Instead of writing the denominator, a point is placed on the left of the numerator.

Thus the fraction $\frac{5}{10}$ is written ·5

$\frac{25}{100}$ „ ·25

$\frac{47}{1000}$ „ ·047

$5\frac{3}{1000}$ „ 5·003

The notation adopted for ordinary numbers may be extended to decimals.

Thus 111·11 =1 hundred + 1 ten + 1 unit + 1 tenth + 1 hundreth.

27·403=2 tens + 7 units + 4 tenths + 0 hundredths + 3 thousandths.

Ciphers on the right of decimals make no alteration in their value; for $\cdot 7 = \frac{7}{10}$, $\cdot 70 = \frac{70}{100} = \frac{7}{10}$, &c.

Ciphers on the left diminish the value in a ten-fold degree; for $\cdot 3 = \frac{3}{10}$, $\cdot 03 = \frac{3}{100}$, &c.

ADDITION OF DECIMALS.

RULE.

Place the numbers so that the decimal *points* may be exactly under each other.

Ex. Add together 4·87, ·004, 278·6, 48, and 32·9874.

```
  4·87
   ·004
278·6
 48·
 32·9874
---------
364·4614  Ans.
---------
```

This rule is the same in principle as that of addition of common numbers.

Examples.

Add together the following quantities:—

1. 6741·583, ·0154967, ·0003872, 100·7259, 531·6741, and 1569·004. Ans. 8943·0028839.
2. 50·6715, 9·873021, 7000·59, ·003268, 5439·005, and 1989·35. Ans. 14489·492789.
3. 9965·321, 67345·28, ·35041, 3695·008, ·000953, and ·876549. Ans. 81006·836912.
4. 7549·8, 597·699, 8754·6, 989·769, 675·00001, and 39·9. Ans. 18606·76801.
5. 3214·065, 39·9054, 598732·0, 9875·936, ·596734, and 60754·9.

6. 96·001, ·3615, ·001, 5948·, 7659·67, 98757·6, and 8·999.

7. 987654·3978, 9888741·004, 366·009, ·008754, 6950·3877, 9·889766759.

8. 9·$\dot{9}$, 87650·0$\dot{8}\dot{5}\dot{4}$, 95·0$\dot{6}$, 78954·22$\dot{3}$, 9543·00095, 5·439998.

In the last example the points placed over some of the figures indicate that these figures, and any between them, must be repeated; thus ·$\dot{9}$ = ·9999 &c. to infinity, ·0$\dot{8}\dot{5}\dot{4}$ = ·0854854854 &c., ·0$\dot{6}$ = ·060606 &c., ·22$\dot{3}$ = 223333 &c. In general these need not be carried farther than to six places from the decimal point.

SUBTRACTION OF DECIMALS.

RULE.

Place the less number under the greater, so that the decimal points may be exactly under each other.

Ex. Take 143·4682 from 8431·27.

8431·27
143·4682

8287·8018 Ans.

This rule is the same in principle as that of subtraction of common numbers.

Examples.

1. Take 50654·012 from 86509·001. Ans. 35854·989.
2. Take 23567·0056 from 24678·0079. Ans. 1111·0023.
3. Take 1325·059 from 56498·561. Ans. 55173·502.
4. Take 7563210·0034 from 9506745·32159. Ans. 1943535·31819.
5. Take 304567·06932 from 543976·6789.
6. Take 95·000678 from 789·006.
7. Take 650·398749 from 1009·39005.
8. Take 854036·5$\dot{9}\dot{0}$ from 999887·50$\dot{6}$.

MULTIPLICATION OF DECIMALS.

RULE.

Multiply as in whole numbers.

Point off from the right of the product as many figures as there are decimal places in both factors.

Ex. 1. Multiply 42·108 by 1·42.

42·108
1·42

84216
168432
42108

59·79336 product.

Proof by fractions.

$$42{\cdot}108 \times 1{\cdot}42 =$$

$$42\frac{108}{1000} \times 1\frac{42}{100} = \frac{42108}{1000} \times \frac{142}{100} =$$

$$\frac{5979336}{100000} = 59\frac{79336}{100000} = 59{\cdot}79336.$$

Here there are 5 decimal places in the two factors, namely, 3 in the multiplicand, and 2 in the multiplier; therefore we mark off 5 from the right of the product.

Ex. 2. Multiply ·000434 by 50510.

·000434
50510

4340
2170
2170

21·921340 product.

Here there are 6 decimal places in the multiplicand and none in the multiplier, therefore we point off 6.

Ex. 3. Multiply ·064 by ·0085.

·064
·0085

320
512

·0005440 product.

Here there are 7 decimal figures in the two factors, and having only 4 figures in the product, we prefix 3 ciphers to make up the deficiency.

Examples.

Find the product of the following quantities.

1. 503·52 × ·005, and prove it by fractions. Ans. 2·5176.
2. 3·76421 × 5·005. Ans. 18·83987105.
3. ·00049 × ·02, and prove it by fractions. Ans. ·0000098.
4. 560·3256 × ·56003. Ans. 313·799145768.
5. 34·00096 × 4·369. Ans. 148·55019424.
6. 79654·86 × 210·6.
7. 4·096085 × ·0009.
8. ·97563 × ·005838.
9. 8765·0001 × 4·3501.
10. ·900566 × ·0509.

DIVISION OF DECIMALS.

RULE.

Divide as in whole numbers, attaching decimal ciphers to the dividend when necessary. There must always be at least as many decimal figures in the dividend as in the divisor.

Point off from the right of the quotient as many decimal places as those in the dividend exceed those in the divisor.

Ex. 1. Divide 1·44 by ·012.

·012)1·440

120 Ans.

Proof by fractions.

$$1{\cdot}44 \div {\cdot}012 = 1\frac{44}{100} \div \frac{12}{1000} =$$

$$\frac{144}{100} \div \frac{12}{1000} = \frac{\overset{12}{\cancel{144}}}{\cancel{100}} \times \frac{\cancel{1000}}{\cancel{12}} = 120.$$

Ex. 2. Divide 51·84 by 3·6

$$\begin{array}{r} 3{\cdot}6\,)\,51{\cdot}84\,(\,14{\cdot}4 \\ 36 \\ \hline 158 \\ 144 \\ \hline 144 \\ 144 \\ \hline \end{array}$$

When the division will not terminate, carry it on, in general, until there will be 5 or 6 decimal figures in the quotient.

Examples.

Ex. 1. Divide ·08289 by ·03; and 71·864 by ·00013.
Ans. 2·763, and 552800.

2. 17000 ÷ 13·75; and 2·975984 ÷ 32·56.
Ans. 1236·36, and ·0914.

3. 56·25 ÷ ·0045; and 5·43968 ÷ 35·6.
Ans. 12500, and ·1528.

4. 48 ÷ ·04; 12 ÷ 1728; and 4·93397 ÷ ·011; proving the first by fractions.

5. 90·9 ÷ ·045; 101·6064 ÷ ·1008; and 1·96 ÷ 4·2; proving the first by fractions.

REDUCTION OF DECIMALS.

To reduce a common fraction to a decimal fraction.

RULE.

Affix decimal ciphers to the numerator, and divide by the denominator.

Ex. Reduce $\frac{3}{8}$, $\frac{2}{9}$, $\frac{1}{11}$, and $\frac{7}{12}$ severally to decimals.

$\frac{3}{8} = \frac{3{\cdot}000}{8} = {\cdot}375$, a terminating decimal.

$\frac{2}{9} = \frac{2{\cdot}0000}{9} = {\cdot}2222$ &c. $= {\cdot}2$, a recurring decimal.

$\frac{1}{11} = \frac{1{\cdot}0000}{11} = {\cdot}0909$ &c. $= {\cdot}0\dot{9}$, a recurring decimal.

$\frac{7}{12} = \frac{7{\cdot}00000}{12} = {\cdot}58333$ &c. $= {\cdot}58\dot{3}$, a mixed recurring decimal.

Examples.

Reduce the following fractions to decimals.

1. $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{4}$, $\frac{2}{5}$, and $\frac{3}{500}$. Ans. ·5, ·25, ·75, ·4 and ·006.
2. $\frac{13}{16}$, $\frac{17}{1250}$, $\frac{1}{12}$, $\frac{1}{63}$, and $\frac{7}{9}$.

Ans. ·8125, ·0136, ·08$\dot{3}$, ·01587$\dot{3}$, and ·$\dot{7}$.

3. $\frac{4}{50}$, $\frac{6}{110}$, $\frac{13}{128}$, $\frac{7}{8}$, $\frac{1}{81}$, and $\frac{4}{99}$.
4. $\frac{5}{36}$, $\frac{10}{11}$, $\frac{3}{8}$, $\frac{2}{75}$, $\frac{2}{300}$, and $\frac{156}{999}$.

To reduce a quantity to an equivalent decimal of higher name.

RULE.

Place the parts one under another, and opposite to them the numbers which will reduce them to the next higher name.

Ex. 1. Reduce 1*s.* 5¾*d.* to the decimal of a pound.

4	3·00
12	5·75000
20	1·47916

·0739583 £. Ans.

Proof by contrary process.

£
·0739583
20

s. 1·4791666
12

d. 5·7499999
4

f. 2·9999999

Ex. 2. Reduce 3 qrs. 2 nls. to the decimal of an Eng. ell, and $\frac{1}{2}d.$ $\frac{4}{5}f.$ to the decimal of half a crown.

		Proof.			Proof. Crown.
4	2·0	E. ell.	5	4·0	·011666
5	3·5	·7	4	2·8	60
		5	12	·70	
	·7 Eng. ell	—	5	·058$\dot{3}$	·699999
		qr. 3·5			4
		4		·011$\dot{6}$	
					f. 2·799999
		nl. 2·0			5
					3·999999

Examples. To be proved by contrary process.

1. Reduce 8*s.* 7½*d.* to the decimal of a pound, and 1 qr. 14 lb. to the decimal of a cwt.

Ans. ·165625 £., and ·375 cwt.

2. Reduce 4½*d.* to the decimal of a crown, and 7 oz. 4 dwt. to the decimal of a pound Troy.

Ans. ·075 cr. and ·6 lb. Troy.

3. Reduce 4 days 1 hour 7½ minutes to the decimal of a week, and 3 bushels 1 peck to the decimal of a quarter.

4. Reduce 12*s.* 4¾*d.* to the decimal of 10*l.*, and 800 yards to the decimal of a mile.

5. Find the integral value of ·5675*l.*, and of ·953125 mile, and prove each operation.

Ans. 11*s.* 4½*d.*, and 7 fur. 25 p.

6. Find the integral value of ·61 of a pipe of wine; and of ·625 of a shilling; and prove each operation.

Ans. 1 hhd. 13 gal. 3 qt.; and 7½*d.*

PROPORTION OF DECIMALS.

Ex. 1. If a person pay $\frac{5}{8}$ £ for ·6 of an Eng. ell, what must he pay for $\frac{9}{10}$ of a yard.

$\frac{5}{8}$ £ $= \frac{5{\cdot}000}{8} =$ ·625 £, ·6 Eng. ell $= \frac{{\cdot}6 \times 5}{4} = \frac{{\cdot}30}{4} =$ ·75 yard.

yd.		yd.		£.
·75	:	·9	::	·625
				·9

$$\cdot 75 \overline{)\cdot 5625} \left(\cdot 75 £ \right.$$

525	20
375	15·00s.
375	

Ans. 15s.

Work out the examples in Proportion of Fractions by decimals.

What is the difference between a common or vulgar fraction and a decimal fraction?

What decimals are equal to $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{4}$, $\frac{1}{8}$, $\frac{7}{8}$, $\frac{1}{3}$? What common fractions are equal to ·4, ·45, ·06, ·015, 2·2, 10·01, $\dot{6}$? What effect have ciphers on the right of a decimal? On the left?

Ex. 1. Express as a decimal fraction 13 + 3 thousands + 5 millionths.

2. Express as a vulgar fraction 7·002707.

Perform the following operations, and give all fractional answers in their lowest terms.

3. ·10000123 + 31·1 + 117·154 + 2343·008 − 92·2624123.

4. ·013 × 1·6 × ·007 × 3·05.

5. 1 ÷ ·013 to 3 places of decimals.

6. Reduce $\cdot 1\dot{3}\dot{6}$ to a vulgar fraction.

7. $\frac{1}{2} + \frac{1}{3} + \frac{7}{8} + \frac{5}{12}$.

8. $\frac{2}{3} \times 1\frac{1}{7} \times 4\frac{3}{8} \times 2\frac{2}{5} \times 2\frac{3}{5}$.

9. $\frac{15\frac{1}{2}}{7\frac{3}{5}}$.

10. $\frac{7}{54}$ of 518*l.* 10*s.*

11. Multiply $3\frac{3}{20}$ by $15\frac{5}{7}$; divide $\frac{2}{3\frac{3}{4}}$ by $\frac{2\frac{3}{4}}{3}$, and subtract the last of these results from the first.

12. Reduce 3 qrs. 21 lb. to the decimal of a cwt., and $\frac{3}{8}$ of $\frac{11}{12}$ of $\frac{16}{25}$ to a decimal.

13. A can do a piece of work in ten days, and B in fifteen days; in how many days will they do the same, both working together?

To find the exact value of a repeating decimal.

The sum of an infinite series in geometrical progression, when the ratio r is a proper fraction, is $\frac{a}{1-r}$, a being the first term of the progression.

Ex. 1. Find the vulgar fraction equivalent to the decimal $\cdot\dot{4}\dot{6}\dot{6}$.

$$\cdot\dot{4}\dot{6}\dot{6} = \frac{4}{10} + \frac{6}{100} + \frac{6}{1000} + \frac{6}{10000} + \&c.$$

Omitting the $\frac{4}{10}$ for the present,

$$a = \frac{6}{100},\ r = \frac{1}{10}.$$

$$\therefore s = \frac{a}{1-r} = \frac{\frac{6}{100}}{1-\frac{1}{10}} = \frac{6}{100-10} = \frac{6}{90}$$

$$\therefore \cdot\dot{4}\dot{6}\dot{6} = \frac{4}{10} + \frac{6}{90} = \frac{36}{90} + \frac{6}{90} = \frac{42}{90} = \frac{7}{15}.$$

Ex. 2. Find the value of the decimal $\cdot 135135$ &c. $= \frac{135}{1000} + \frac{135}{1000000} + \&c.$

$$a = \frac{135}{1000},\ r = \frac{1}{1000}$$

$$\therefore s = \frac{a}{1-r} = \frac{\frac{135}{1000}}{1-\frac{1}{1000}} = \frac{135}{1000-1} = \frac{135}{999} = \frac{45}{333} = \frac{5}{37}.$$

Ex. 3. What is the exact value of the decimal $\cdot 2010101$ &c.

$$\cdot 2010101 = \frac{2}{10} + \frac{1}{1000} + \frac{1}{10000} + \&c.$$

Omitting the $\frac{2}{10}$ for the present,

$$\therefore s = \frac{a}{1-r} = \frac{\frac{a}{1000}}{1-\frac{1}{100}} = \frac{1}{1000-10} = \frac{1}{990}$$

$$\therefore \cdot 2010\dot{1}\dot{1} = \frac{2}{10} + \frac{1}{990} = \frac{198}{990} + \frac{1}{990} = \frac{199}{990}.$$

Find the exact value of the following decimals:
(1) $\cdot 5\dot{3}\dot{3}$, (2) $\cdot 141414$ &c., (3) $\cdot 403403$ &c., (4) $6\cdot 11\dot{1}$, (5) $25\cdot 040\dot{4}$, (6) $\cdot 480101$, (7) $3\cdot 02001001$, (8) $\cdot 2\dot{2}$, (9) $\cdot 5\dot{7}$, (10) $\cdot 572357235$ &c., (11) $\cdot 0\dot{4}$, (12) $\cdot 07\dot{5}\dot{3}$, (13) $\cdot 47\dot{3}$, (14) $\cdot 3\dot{5}\dot{4}$, (15) $\cdot \dot{1}2\dot{5}$.

INVOLUTION.

Involution is raising a number to any required *power* by repeated multiplication.

Thus, $3 \times 3 = 3^2 = 9$ is the square or 2nd power of 3;
$2 \times 2 \times 2 = 2^3 = 8$ is the cube or 3rd power of 2;
$5 \times 5 \times 5 \times 5 = 5^4 = 625$ is the 4th power of 5, &c.

The number denoting the power is called the *index* or exponent of that power; thus, 7^5 denotes the 5th power of 7, the index being 5.

A power of a fraction is obtained by raising its numerator and denominator severally to the required power; thus, the cube of $\frac{3}{4} = \frac{3^3}{4^3} = \frac{27}{64}$: the square of $\frac{1}{9} = \frac{1^2}{9^2} = \frac{1}{81}$.

Ex. 1. Make a table of the squares of the first twelve numbers.

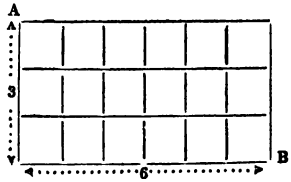
2. Make a table of the cubes of the first nine numbers.

3. Find the square of 1·25; and the cube of 1·04.
Ans. 6·25; and 1·124864.

4. Find the fifth power of $\frac{2}{3}$, the fourth power of 1·01, and the eighth power of 2.

The area of any parallelogram is equal to the product of its *base* and *perpendicular* height.

Thus, if the base of the rectangle AB be 6 inches, and its height 3 inches, its area will be $6 \times 3 = 18$ square inches.



F 3

EVOLUTION.

Evolution is the extracting of the root of any number. It is the reverse of Involution.

Roots are denoted by $\surd$ or by a fractional index; thus,

$\surd\overline{4}$ or $4^{\frac{1}{2}} = 2$, denotes the square root of 4;

$\sqrt[3]{\overline{64}}$ or $64^{\frac{1}{3}} = 4$, denotes the cube root of 64.

The rule for extracting the square root is derived from the formula $(a + b)^2 = a^2 + (2a + b)b$.

Ex. 1. Extract the square root of 1225.

$$\begin{array}{rl} & \dot{1}2\dot{2}\dot{5} \text{ (35 the root.} \\ & 9 \\ & \hline \\ 65) & 325 \\ & 325 \\ & \hline \\ & \cdots \end{array}$$

We first place a point over the *units* figure, and then over every alternate figure, right and left.

In the above example the process is as follows. The greatest square in 12 is 9; the 9 is set down under the first period, and its root 3 on the right; the root is doubled for a divisor; the next figure of the root is found by dividing 325 (omitting the last figure 5) by the divisor 6, and the root figure 5 is placed in the root's place, and also annexed to the divisor; 65 is then multiplied by the root figure 5, and we subtract.

If there were a remainder, we should annex to it *two* decimal ciphers, double the 35, for a divisor, and proceed as before.

The process is precisely similar to the following extraction of the square root of $(a + b)^2$ or $a^2 + (2a + b)b$.

$$\begin{array}{rl} & a^2 + 2ab + b^2 (a + b \\ & a^2 \\ & \hline \\ 2a + b) & 2ab + b^2 \\ & 2ab + b^2 \\ & \hline \\ & \cdots \end{array}$$

Ex. 2. Extract the square root of $213\frac{5}{24}$, of $5\frac{2}{9}$, and of $1\frac{2}{3}$.

$$\sqrt{213\frac{5}{24}} = \sqrt{\frac{5329}{25}} = \frac{\sqrt{5329}}{\sqrt{25}} = \frac{73}{5} = 14\tfrac{3}{5},$$

$$\sqrt{5\frac{2}{9}} = \sqrt{\frac{47}{9}} = \frac{\sqrt{47}}{\sqrt{9}} = \frac{6{\cdot}85565}{3} = 2{\cdot}28522 \text{ nearly}$$

$$\sqrt{1\tfrac{2}{3}} = \sqrt{1{\cdot}666666} \text{ &c.} = 1{\cdot}2909 \text{ nearly.}$$

The rule for extracting the cube root is derived from the formula $(a + b)^3 = a^3 + (3a^2 + 3ab + b^2)b$.

Ex. 1. Extract the cube root of 13824.

$$\begin{array}{rl} & \dot{1}3\dot{8}2\dot{4} \text{ (24 the root.} \\ & 8 \\ \hline 2^2 \times 3 = 12 \;) & 5824 \\ \hline 4^3 = & 64 \\ 4^2 \times 2 \times 3 = & 96 \\ 4 \times 12 = & 48 \\ \hline & 5824 \\ \hline & \ldots\ldots \end{array}$$

We first place a point over the *units* figure, and then over every third figure, right and left.

In the above example the process is as follows. The greatest cube in 13 is 8; the 8 is set down under the first period, and its cube root 2 on the right.

For a divisor, we multiply the square of the root by 3.

For a subtrahend, we cube the last figure of the root, multiply the square of the last figure by whatever is before it, and also by 3, multiply the last figure by the divisor, and add these three results together.

If there were a remainder, we should annex to it *three* decimal ciphers, multiply the square of the root (24) by 3, and proceed as before.

The process is precisely similar to the following extraction of the cube root of $(a+b)^3$ or $a^3 + 3a^2b + 3ab^2 + b^3$.

$$a^3 + 3a^2b + 3ab^2 + b^3 \ (a + b$$
$$a^3$$

$a^2 \times 3 = 3a^2$) $3a^2b + 3ab^2 + b^3$

$b^3 =$ $+ b^3$

$b^3 \times a \times 3 =$ $+ 3ab^2$

$b \times 3a^2 =$ $3a^2b$

$3a^2b + 3ab^2 + b^3$

Examples.

1. Extract the square roots of 59049, 94249, 29·41275, and 75600. Ans. 243, 307, 5·4233, and 274·95.

2. Extract the square roots of $2\frac{7}{9}$, $\frac{169}{3481}$, $15\frac{4}{5}$, 5, ·043, and ·004.

3 Extract the cube roots of 830584, 218167208, 47·8475, and ·09375. Ans. 94, 602, 3·6303, and ·45428.

4 Extract the cube roots of 11·71875, ·000001728, $42\frac{7}{8}$, $5\frac{1}{64}$, 2, and 500·3737, &c.

5. Extract the square and cube roots of 3, ·012, 30·625, $\frac{3}{7}$ and 20.

6. A garden of square form contains 1548·124 square yards; find the length of its side. Ans. 39·346 yards.

7. Find the side of a square field which contains exactly an acre. Ans. 69·5701 yards.

8. The sides of two squares are 9 and 12 respectively; find the side of a third square whose area is equal to the sum of the areas of the other two. Ans. 15 feet.

9. The base of a right-angled triangle is 15 feet, and the perpendicular 8 feet; find the length of the hypothenuse. Ans. 17 ft.

10. The hypothenuse being 14 ft. 2 in., and the perpendicular 11f. 4in., what is the base? Ans. 8 ft. 6 in.

11. A rectangular field measures 70 yards in length and $52\frac{1}{2}$ yards in breadth; what is the length of a diagonal path path across it? Ans. $87\frac{1}{2}$ yards.

12. A tower stands on the other side of a ditch, 18 feet wide; how high up the tower will a ladder, 25 feet long, reach from this side? Ans. 17·349 ft.

13. The diagonal of a square field is 30·9 chains; find the area of the field, and the cost of fencing it round at 3*s.* per yard.

14. A farmer wishes to build a fold to contain a certain number of sheep; it must either be a square, or a rectangle whose breadth is $\frac{3}{4}$ of its length: which form should he select so as to save expense in building the walls?

15. The solid content of a sphere is 941·25 cubic inches; find the side of a cube of equal solidity.

16. A ball 4 inches in diameter weighs 9lbs.; find the diameter of a similar ball weighing 72lbs.

17. A cistern is 8 feet long, 6 broad, and 5 deep; find the dimensions of a similar cistern to contain 4 times as much.

18. A ship of 480 tons burden is 40 feet long in the keel; what length of keel must a similar ship have to be of 800 tons burden?

19. Five metal cubes, whose edges are 2, 3, and 4 inches respectively, are to be formed into one cube; find the length of its edge and the area of its side.

20. A sphere of metal, whose radius is 3 inches, is to be beaten out into a circular plate $\frac{3}{10}$ inch thick; find the radius of the plate.

DUODECIMALS.

This is a convenient method of ascertaining areas and solid contents when the dimensions are given in feet, inches, parts, &c., decreasing in a twelve-fold proportion.

These are usually marked—feet, $'$, $''$, $'''$, $''''$, &c.; that is, feet, primes, seconds, thirds, fourths, &c.

Ex. 1. Find the surface of a board 8 ft. 7 in. 5 pa. long, and 3 ft. 8 in. 10 pa. broad.

ft.	in.	pa.		
8 .	7 .	5		
3 .	8 .	10		
25 .	10 .	3		
5 .	8 .	11	. 4	
	7 .	2	. 2	. 2
32 .	2′ .	4″	. 6‴	. 2⁗

Ans. 32 sq. ft. 2 sq. in. 4 sq. pa. 6 twelfths, 2 twelfths of twelfths.

The upper line is multiplied first by 3, then by 8, and lastly by 10, dividing continually by 12; thus, 3 times 5 are 15, set down 3 and carry 1; 3 times 7 are 21, 21 and 1 are 22, set down 10 and carry 1; 3 times 8 are 24, 24 and 1 are 25, set down 25 feet.

In multiplying by 8, the 1st remainder 4 is put one place farther to the right, because $\frac{8}{12} \times \frac{5}{144} = \frac{40}{1728} = 40''' = 3'' \,.\, 4'''$.

For similar reasons, the 1st remainder in the next line is put one place still farther to the right.

Examples.

1. Find the area of a rectangle, whose dimensions are 35 ft. 4 in. 6 pa., and 12 ft. 3 in. Ans. 433 sq. ft. 4′ . 1″ . 6‴.

2. Find the area of a marble slab whose sides measure 6 ft. 4 in. and 4 ft. 7 in. Ans. 29 sq. ft. 0 pa. 4 in.

3. Find the price of a slab whose dimensions are 5 ft. 7 in. and 1 ft. 10 in., at 6*s.* per square foot. Ans. 3*l.* 1*s.* 5*d.*

4. Find the expense of paving a court, at $4\frac{3}{4}d.$ per yard, the dimensions being 58 ft. 6 in. and 54 ft. 9 in.
Ans. 7*l.* 0*s.* $10\frac{1}{4}d.$

5. The dimensions of a floor are 12 ft. 5 in. and 10 ft. 1 in. 4 pa.; what will it cost at 5*s.* 6*d.* per yard?

6. Find the solid content of a stone which measures 6 ft. 2 in. in length, 4 ft. 1 in. in breadth, and 2 ft. 6 in. in depth.

7. How many ounces of air are contained in a room 20 ft. long, 12 ft. 3 in. wide, and 11 ft. 4 in. high, a cubic foot of air weighing $1\frac{1}{4}$ oz.; and how many gallons of water would fill it?

What is the difference between Involution and Evolution? What is the square of 3, of 7, of $\frac{1}{2}$, and of ·4? What is the cube of 2, of 4, of $\frac{2}{3}$, of ·1? What is the square root of 16, of 81, of 144, of 1? What is the cube root of 27, of 1000, of 125, of $\frac{1}{8}$? Name the algebraic formulæ on which the rules for extracting square and cube roots depend. How many times do you multiply to raise a number to the tenth power? What is the object of duodecimals? What is the difference between duodecimals and decimals? How would you find the 4th root of a number? the 6th root?

Miscellaneous Examples.

1. A cubical block contains 103823 solid feet; find its superficies.

2. A tank, 6 feet long and 5 feet wide, contains 120 cubic feet of water; find the depth of water.

3. After taking away $\frac{1}{3}$th of a cube whose side is 10 inches, the remaining mass is formed into 8 equal cubes; find the length of one of their edges.

4. If a cistern can be filled in 5 hours by a pipe 6 inches in diameter, how many pipes, 2 inches in diameter, will fill it in the same time.

5. Find the cost of a log of timber 16 ft. 4 in. long, 2 ft. 1 in. broad, and 1 ft. 6 in. thick, at 1*s.* 10*d.* per cubic ft

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SECTION III.

EXCHANGE TABLES.

FRANCE.

12 deniers	1 sou
20 sous	1 livre
3 livres.....	1 écu

Also,

10 centimes	1 décime
10 décimes.....	1 franc

SPAIN.

34 maravedies	1 real
8 reals	1 piastre
4 piastres.....	1 pistole
375 maravedies	1 ducat

PORTUGAL.

400 rees	1 crusado
1000 rees	1 milree

HOLLAND.

16 pennings	1 stiver (2*d.*)
20 stivers	1 florin
2½ florins	1 rix-dollar
6 florins	1 pound (Flemish)

Also,

8 pennings	1 penny	Flem.
12 pence	1 shilling	
20 shillings	1 pound	

GERMANY.

12 phennings.....	1 schilling
16 schillings	1 mark
2 marks	1 dollar of exchange
3 marks	1 rix-dollar

Also,

6 phennings.....	1 penny	Flem.
12 pence	1 shilling	
20 shillings	1 pound	

ITALY*.

12 denari	1 soldo
20 soldi	1 piastre

RUSSIA.

100 copecs	1 ruble
2 rubles	1 ducat

INTEREST.

Interest is what is paid for money lent. *Principal* is money lent. *Rate per cent.* is interest of 100*l.* for 1 year. *Amount* is principal + interest.

When interest is paid only on the money lent, it is *Simple Interest.* When interest is at certain intervals added, and interest again calculated on the amount, it is *Compound Interest.*

Commission and *Brokerage* are an allowance of so much per cent. paid to agents or brokers for buying or selling goods or stock. *Insurance* is a payment of so much per cent. to secure property from losses by fire, shipwreck, &c.

Stocks or *Public Funds* are debts of Government contracted by borrowing money of the public. The interest is called a *dividend.*

* Livres and cents, similar to francs and cents in France, are used in some places in Italy.

RULE.

1. To find the *simple* interest for one year,

$$\frac{\text{principal} \times \text{rate per cent.}}{100}.$$

2. To find the *simple* interest for several years,

$$\frac{\text{principal} \times \text{rate per cent.} \times \text{the years}}{100}.$$

When the time consists of months, weeks, or days, apply Practice or Proportion.

Rule 1 applies also to Commission, Brokerage, Insurance, and Purchasing of Stock.

DISCOUNT.

Discount is an allowance for payment of money before it is due. It is the *difference* between a sum and its present value.

RULE.

Amount of £100 for the given time : the given sum :: £100 : present value.

EQUATION OF PAYMENTS.

Equation of Payments is a method of finding the time at which several payments, due at *different* times, should be paid at once.

RULE.

Multiply each payment by its time, and divide the sum of the products by the sum of the payments.

PARTNERSHIP.

Partnership or *Fellowship* is a method of finding the shares in the profit or loss of several persons having a joint stock.

RULE.

Whole stock : each man's stock :: whole gain or loss : each man's gain or loss.

When the partners employ their stock for different periods, multiply each man's stock by his time, and proceed as above.

ALLIGATION.

Alligation is a method of finding the *rate* or *quantity* of several ingredients to be mixed together.

RULE.

To find the mean rate, multiply each ingredient by its rate, and divide the sum of the product by the sum of the ingredients.

To find the quantity of each ingredient. Set down the prices under each other, with the mean rate in front; link together each one less than the mean with one greater than it, and each one greater with one less; compare each with the mean rate, and write its difference opposite that with which it is linked. These differences will form the answer.

TARE AND TRET.

Tare is an allowance for the weight of the case, bag, &c., in which goods are packed.

Tret is an allowance of 4 lb. in 104 lb. for waste, dust, &c.

Cloff is an allowance of 2 lb. in 336 lb., after deducting Tare and Tret, for loss of weight in selling by retail.

Suttle is the remainder when Tare has been deducted from the gross weight.

Net weight is the remainder after all allowances have been deducted.

Operations in this rule may be performed by Division and Subtraction.

EXCHANGE.

Exchange is the paying or receiving of the money of one place for its equivalent in that of another.

Operations in this rule may be performed by Proportion or Practice.

ARBITRATION OF EXCHANGE.

Arbitration of Exchange is a comparison of the rates of exchange of different places, so as to determine the most advantageous way of drawing bills or transmitting money.

Operations in this rule may be performed by the *Chain Rule*.

CHAIN RULE.

RULE.

Set down the term of demand, then arrange all the other quantities as equations, making each term on the left of the

same name as the one immediately preceding it on the right. Divide the product of all the terms on the right by the product of all those on the left.

Ex. 1. If a person invest in the $3\frac{1}{2}$ per cents. at $92\frac{5}{8}$, including the broker's commission, what interest will he get?

Here, by lending $92\frac{5}{8}$*l.*, he will get $3\frac{1}{2}$*l.* per annum interest;

$\therefore 92\frac{5}{8} : 100 :: 3\frac{1}{2}$: interest required,

or, $741 : 800 :: 3\frac{1}{2}$

$3\frac{1}{2}$

2400
400

741) 2800 (3*l.* 15*s.* 6*d.* $\frac{2}{3}\frac{1}{4}\frac{8}{7}$ per cent. Ans.

Ex. 2. What is the present worth and discount of 400*l.* 15*s.*, at $4\frac{1}{2}$ per cent. for 6 months? or, what sum put out at $4\frac{1}{2}$ per cent. simple interest, for 6 months, will amount to 400*l.* 15*s.*; and what will be the interest of that sum?

	£	s.	
6 m. | $\frac{1}{2}$ |	4	10	= interest of 100*l.* for 12 months.
	2	5	= . . . . . 6 months.
	100		
	102	5	= amount

£	s.	£	s.		£
102	5	400	15	::	100
4		4			
409		160300			

(391*l.* 18*s.* 7*d.* $\frac{2}{4}\frac{3}{0}\frac{3}{9}$.

	£	s.	d.	
	400	15	0	
Ans.	391	18	$7\frac{2}{4}\frac{3}{0}\frac{3}{9}$	= present value.
	18	16	$4\frac{1}{4}\frac{7}{0}\frac{6}{9}$	= discount.

Ex. 3. The cost of 50 gallons of wine is 15*l.*, but $\frac{1}{5}$ is lost by leakage, and 20 gallons are sold at 4*s.* per gallon; at what price per gallon must the remainder be sold to gain 5 per cent. upon the whole?

$50 \div 5 = 10 =$ gals. lost by leakage.
$\therefore 50 - 10 = 40 =$ gals. remaining.
20 gals. at 4*s.* $= 4l. =$ selling price of 20 gals.
$40 - 20 = 20 =$ gals. still remaining to be sold.

Cost. Cost. Selling price.
100*l.* : 15*l.* :: 105*l.* : 15*l.* 15*s.* = selling price of the 40 gals.

$\therefore$ 15*l.* 15*s.* $- 4l. = 11l.$ 15*s.* = selling price of the remaining 20 gals.

$11l.\ 15s. \div 20 = 11s.\ 9d.$ Ans.

Ex. 4. A has flax worth 3*l.* 16*s.* per cwt., which he offers B in barter for linen worth $7\frac{1}{2}d.$ per yard; B agrees to take flax in part, but rates his linen at $8\frac{1}{4}d.$ per yard, and insists on having $\frac{1}{3}$ of that value in cash: what must A charge per cwt. for his flax, in order to be on equal terms with B?

$8\frac{1}{4}d. \div 3d. = 2\frac{3}{4}d. =$ cash A must give B.
$8\frac{1}{4}d. - 2\frac{3}{4}d. = 5\frac{1}{2}d. =$ barter price of linen.
$7\frac{1}{2}d. - 2\frac{3}{4}d. = 4\frac{3}{4}d. =$ real value of that linen.

Real value. Barter price. Real value. Barter price.
$4\frac{3}{4}d.$: $5\frac{1}{2}d.$ :: 3*l.* 16*s.* : 4*l.* 8*s.* Ans.

Ex. 5. Bought 13*l.* 6*s.* 8*d.* worth of fruit at 3*s.* 4*d.* per bushel, part of which, being damaged, was entirely lost: for the rest, which sold at 50 per cent. profit, I received 16*l.*: how many bushels were damaged?

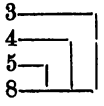
3*s.* 4*d.* : 13*l.* 6*s.* 8*d.* :: 1 bush. : 80 bushels bought.

Selling price. Selling price. Cost.
150*l.* : 16*l.* :: 100*l.* : 10*l.* 13*s.* 4*d.* = cost of the remainder.

13*l.* 6*s.* 8*d.* : 10*l.* 13*s.* 4*d.* :: 80 bush. : 64 bush. = remainder.

$\therefore 80 - 64 = 16$ bushels damaged. Ans.

Ex. 6. How many ounces of tea at 3*d.*, 4*d.*, 5*d.*, and 8*d.* per oz., must be mixed together to form a mixture worth 6*d.* an ounce?



3	2	at 3*d.* = 6*d.*
4	2	at 4*d.* = 8
5	2	at 5*d.* = 10
8	3 + 2 + 1 = 6	at 8*d.* = 48
	12 oz. at	72*d.*
or	1 oz. at	6*d.* Proof.

Ans. 6 oz. at 8*d.*, and 2 oz. of each of the others.

Ex. 7. If 27 sheep are worth 3 oxen, and 24 oxen are worth 18 cows, and 42 cows are worth 36 horses, how many sheep are worth 40 horses?

The term of demand is 40 horses;

40 horses
36 horses = 42 cows
18 cows = 24 oxen
3 oxen = 27 sheep

$$\frac{40 \times \overset{14}{\cancel{42}} \times \overset{4}{\cancel{24}} \times \overset{3}{\cancel{27}}}{\underset{3}{\cancel{36}} \times \underset{3}{\cancel{18}} \times \cancel{3}} = 560 \text{ sheep. Ans.}$$

Examples.

1. Find the simple interest and amount of 355*l.* 15*s.*, for 4 years, at 4 per cent. per annum.

Ans. 56*l.* 18*s.* $4\frac{3}{4}$*d.*, and 412*l.* 13*s.* $4\frac{3}{4}$*d.*

2. Find the simple interest and amount of 204*l.* 16*s.*, for $5\frac{1}{2}$ years, at 3 per cent. per annum.

3. The interest of 319*l.* 6*s.*, for $5\frac{3}{4}$ years, was 68*l.* 14*s.* $9\frac{1}{2}$*d.*; find the rate per cent. Ans. $3\frac{3}{4}$.

4. 170*l.* amounted to 182*l.* 15*s.*, at 5 per cent.; find the time.

5. Find the amount at compound interest of 217*l.*, forborne $2\frac{1}{4}$ years, at 5 per cent., interest payable quarterly.

Ans. 242*l.* 13*s.* $4\frac{1}{2}$*d.*

6. Find the compound interest of 250*l.* for 3 years, the interest being payable yearly.

7. If I pay an agent $3\frac{1}{4}$ per cent. for selling goods to the amount of 240*l.* 16*s.* 8*d.*, what is his commission?

Ans. 7*l.* 16*s.* $6\frac{1}{2}$*d.*

8. If a factor's charge be 5*s.* 6*d.* per cent., what is his commission on the sale of goods for 2460*l.* 16*s.*?

9. The stock-broker's charge being $\frac{1}{8}$*l.* per cent., what is his commission on the sale of 573*l.* 16*s.* $10\frac{1}{2}$*d.* stock?

Ans. 14*s.* 4*d.* $\frac{42}{80}$.

10. What must be paid to a stock broker for buying 360*l.* stock in the 3 per cents. reduced, the commission being $\frac{1}{8}$*l.* per cent.?

11. A cargo valued at 3561*l.* 11*s.* $7\frac{1}{2}$*d.* being insured at $8\frac{1}{3}$ per cent., what is the insurance? Ans. 296*l.* 15*s.* $11\frac{1}{2}$*d.* $\frac{1}{2}$.

12. A house and furniture being insured for 845*l.*, at 10*s.* per cent., what is the insurance?

13. What money must be paid for 7280*l.* stock in the 3 per cent. consols., at $84\frac{7}{8}$*l.* per cent.; that is, when $84\frac{7}{8}$*l.* money will buy 100*l.* stock? Ans. 6178*l.* 18*s.*

14. What is the purchase money for 1558*l.* 10*s.* stock in the $3\frac{1}{4}$ per cents., at $95\frac{3}{4}$?

15. What amount of stock in the 3 per cent. consols can I purchase for 4734*l.*, the price being $65\frac{3}{4}$. Ans. 7200*l.*

16. How much stock can be bought for 1009*l.* 10*s.*, when the price is $84\frac{1}{8}$*l.*; that is, when 100*l.* stock can be bought for $84\frac{1}{8}$*l.*?

17. What interest of money arises from investing in a 5 per cent. stock, at 95? Ans. $5\frac{5}{19}$ per cent.

18. What income will arise from 2400*l.* invested in the 3 per cent. reduced annuities, at $89\frac{3}{8}$ per cent?

19. The 3 per cents. being at $92\frac{1}{8}$, and the $3\frac{1}{2}$ per cents. at $93\frac{1}{2}$, into which stock is it most advantageous to buy?

20. A person transfers 9000*l.* stock from the $3\frac{1}{4}$ per cents. at $87\frac{3}{4}$ to the 3 per cents. at $87\frac{1}{8}$; find the alteration in his income.

21. What is the discount of 690*l.* 3*s.* 9*d.*, due 9 months hence, at 3 per cent. per annum? Ans. 15*l.* 3*s.* 9*d.*

22. Find the present value and discount of 360*l.* 10*s.*, due in 15 months, at $4\frac{1}{2}$ per cent.

23. What is the discount of 45*l.* 12*s.* 6*d.* for 56 days, at 5 per cent. per annum? Ans. 7*s.*

24. Required the present worth and discount of a 3 months

bill of exchange drawn on the 7th of April, 1849, allowing the usual 3 days grace.

25. A debt is to be paid thus; $\frac{1}{4}$ at 2 months, $\frac{1}{8}$ at 3 months, $\frac{1}{8}$ at 4 months, $\frac{1}{8}$ at 5 months, and the rest at 7 months; what is the equated time to pay the whole *at once?*

Ans. 4m. 18da.

26. A debt is to be paid thus; 100*l.* in 9 months, and 500*l.* in a year and 3 months: find the equated time.

27. A, B, C, D form a joint stock thus; A puts in 180*l.*, B 240*l.*, C 350*l.*, and D 430*l.*; they gain 3600*l.*: what is each man's share of this gain? Ans. A 540*l.*, B 720*l.*, C 1050*l.*, D 1290*l.*

28. Three merchants, A, B and C, enter into partnership; A contributed to the stock 2400*l.*, B 1060*l.*, and C 1000*l.*; they lose 2560*l.* 10*s.*: find each man's share of this loss.

29. Three graziers farm a piece of land for 60*l.* 10*s.*; A put in 5 sheep for $4\frac{1}{2}$ months, B 8 for 5 months, C 9 for $6\frac{1}{2}$ months: how much of the rent must each pay?

Ans. A 11*l.* 5*s.*, B 20*l.*, C 29*l.* 5*s.*

30. A and B form a joint stock; A advanced 50*l.* for 4 months, and B 60*l.* for 5 months: what is each person's share of the gain or loss?

31. If three sorts of gunpowder be mixed together, viz., 50 lb. at 12*d.*, 44 lb. at 9*d.*, and 26 lb. at 8*d.* a pound: what is the mixture worth per lb.? Ans. $10\frac{1}{30}d.$ per lb.

32. If 5 gallons of wine at 12*s.*, 6 gallons at 10*s.*, 4 gallons at 16*s.*, and 7 gallons at 11*s.* a gallon be mixed together, what is the value of a gallon of the mixture?

33. How many bushels of corn at 2*s.* 6*d.*, 3*s.* 8*d.*, 4*s.*, and 4*s.* 8*d.* per bushel, must be mixed together that the mixture may be worth 3*s.* 10*d.* per bushel?

Ans. 2 at 2*s.* 6*d.*, 3 at 3*s.* 8*d.*, 3 at 4*s.*, and 3 at 4*s.* 8*d.*

34. How much sugar at 4*d.*, 6*d.*, 8*d.*, and 9*d.* per lb., must be mixed together to make a mixture worth 7*d.* per lb.?

35. What is the net weight of 32 cwt. 3 qr. 12 lb. of sugar, tare 14 lb. per cwt., and tret 4 lb. in 104 lb.?

Ans. 27 cwt. 2 qr. $16\frac{2}{13}$ lb.

36. What is the value of the net weight of 10 bags of yarn, each 2 cwt. 1 qr. 12 lb., allowing 24 lb. per cwt. for weight of package, and 4 lb. in 104 lb. for tret?

37. What is the net weight of 6 hhds. of tobacco, each 9 cwt. 1 qr. 14 lb., tare 3 qr. 18 lb. per hhd., tret 4 lb. in 104, and cloff 2 lb. in 336? Ans. 48 cwt. 2 qr. $4\frac{18\frac{1}{2}}{112}$ lb.

38. Find the net weight of 25 cwt. 1 qr. of raisins, tare 16 lb. per cwt., tret and cloff as usual, and find the value of the net weight at $6\frac{1}{2}d.$ per lb.

39. How many francs are equivalent to 675*l.* 18*s.* 3*d.* sterling, when 23 francs 15 cents are equivalent to 1*l.*?

Ans. 15647·374 francs.

40. How many pounds sterling are equivalent to 4364 marks 9 schillings, the exchange being 13 marks $10\frac{1}{4}$ schillings per 1*l.*?

41. When the course of exchange between London and Petersburg is $37\frac{1}{2}d.$ per ruble, how many rubles and copecs are equivalent to 1196*l.* 5*s.* $9\frac{1}{2}d.$ Ans. 7656 rub. 25 cop.

42. Reduce 7420*l.* into florins and kreutzers, at 10 florins 2 kreutzers per 1*l.* sterling.

43. If 80 lb. at London are equal to 72 lb. at Amsterdam, and 45 lb. at Amsterdam equal to 58 lb. at Dantzic, how many pounds at London are equal to 130 lb. at Dantzic?

Ans. $112\frac{2}{3}\frac{2}{9}$ lb.

44. The rate of exchange between Paris and Amsterdam being 211 francs for 100 florins, and between London and Amsterdam 12 florins 15 cents for 1*l.* sterling, what is the rate of exchange between London and Paris, that is, how many francs are worth one pound sterling?

Miscellaneous Examples.

1. *Exchange.* Exchange 554*l.*13*s.* 7*d.* sterling, into francs, at 25 francs $41\frac{1}{4}$ centimes per 1*l.* sterling.

2. *Partnership.* A, B, and C join in a speculation, by which they gain 165*l.*; what should each receive, A having advanced 80*l.*, B 75*l.*, and C 120*l.*?

3. *Profit and Loss.* By selling wine at $1\frac{1}{2}$ guinea per doz., $12\frac{1}{2}$ per cent. was lost; what was the prime cost per dozen?

4. *Barter.* How much cheese at 21*s.* 6*d.* per cwt. together with 17*l.* in cash, must A give B for 16 pieces of cloth at 3*l.* 15*s.* per piece?

5. *Equation of Payments.* A debt is to be paid as follows, namely; $\frac{1}{3}$ in three months, $\frac{1}{3}$ in five months, and the rest in eight months; when should the whole be paid *at once?*

6. *Discount.* What is the present worth of 399*l.* 13*s.* 4*d.* payable in 73 days, at 5 per cent.?

7. *Decimals.* From $\frac{3}{8}$ of a guinea take $\frac{3}{4}$ of 7*s.* 6*d.*, and reduce the result to the decimal of a moidore.

8. Multiply 2·004 by 84·375, and divide the product by ·167.

9. *Fractions.* Reduce 1 qr. 13 lb. $\frac{1}{2}$ oz. to the fraction of a ton, and find the value of $\frac{2}{3}$ of $\frac{5}{8}$ of 13*s.* 4*d.*

10. Find the difference between $1\frac{2}{3}$ and $\frac{7}{8}$, and the sum of $2\frac{1}{6}$ and $13\frac{3}{4}$.

11. Reduce $\frac{3}{5}$, $\frac{1}{6}$ and $\frac{2}{3}$ to equivalent fractions with the least common denominator.

12. *Stocks.* What income will arise from investing 1000*l.* in the 3 per cents. at $93\frac{1}{4}$?

13. *Commission.* What is the commission on 789*l.* 16*s.* 8*d.*, at $1\frac{1}{4}$ per cent.?

14. *Insurance.* What must be paid for insurance of a house worth 681*l.* 5*s.*, at 5*s.* 3*d.* per cent.?

15. *Interest.* At what rate per cent. will 250*l.* amount to 300*l.* 12*s.* 6*d.* in $4\frac{1}{2}$ years?

16. *Tare and Tret.* What is the net weight of 20 bales of silk, each 1 cwt. 2 qrs. 13 lb., tare 18 lb. per bale?

17. *Practice.* 12 qr. 3 bush. 3 pecks, at 2*l.* 2*s.* 8*d.* per quarter.

18. *Proportion.* If a cwt. of butter cost 5*l.* 2*s.* 8*d.*, what is the value of $75\frac{1}{2}$ lb.?

19. If a pendulum vibrates 4 times in 5 seconds, how many times will it vibrate in a week?

EXAMINATION PAPER. No. 1.

FIRST FOUR RULES (SIMPLE).

1. Write down in figures ten millions seventy thousand and fifty.

2. Write down in words at length, 1078400 and 50270106.

3. Write down in figures, as one number, seventy millions seventy thousand seventy hundred and seventy.

4. A straight line, AB, is divided into five parts; the distance from A to E is 35 inches, from E to D 27 inches, from D to C 40 inches, from C to F 19 inches, and from F to B 53 inches: what is the distance from A to B?

Ans. 174 inches

5. Add together the following numbers: 467431, 22816, 24, 538 and 1640081, and subtract their sum from that of 213864, 701253 and 7. Ans. Sum, 2130890; diff., 1215766.

6. Give a full explanation of every step in the process of addition and subtraction in the last example.

7. Multiply together 43701 and 269, and divide the product by 586. Ans. Product, 11755569; quotient, $20060\frac{409}{586}$.

8. Explain the rule for multiplication, by the last example.

9. Divide 391784671 by 343, by short division, and show the true remainder. Ans. 1142229 and 124 rem.

10. Divide 74958467 by 84600, and 3759484 by 1200. Ans. 886 and 2867 rem.; 3132 and 1084 rem.

EXAMINATION PAPER. No. 2.

COMPOUND RULES.

1. There are due to me the following sums: 18*l.* 3*s.* 4*d.*, 19*l.* 6*s.* $7\frac{1}{2}d.$, 133*l.* 15*s.* $2\frac{3}{4}d.$, and a half share of 293*l.* 17*s.* 0*d.*, out of which I have to pay 190*l.* 14*s.* $7\frac{1}{2}d.$ and 76*l.* 13*s.* 3*d.*: how much will remain due to me? Ans. 50*l.* 15*s.* $9\frac{3}{4}d.$

2. What is the amount of the following items: $54\frac{1}{4}$ lbs. at 7*s.* $4\frac{1}{2}d.$ per lb., 241 yards at $7\frac{1}{2}d.$ per yard, and 512 pieces at 1*s.* $4\frac{3}{4}d.$ a piece? Ans. 62*l.* 6*s.* $7\frac{1}{2}d.$ 5.

3. Which is the larger sum, 14*l.* 11*s.* $7\frac{1}{2}d.$, or 13861 farthings, and by how much? Work this question in two different ways. Ans. 2*s.* $10\frac{1}{4}d.$

4. Add together 41*l.* 18*s.* $10\frac{1}{4}d.$, 64*l.* 19*s.* $5\frac{1}{2}d.$, 7*l.*, 3*l.* 5*s.* $1\frac{1}{2}d.$, 17*l.* 18*s.* $0\frac{1}{4}d.$, and 94*l.* 18*s.* $8\frac{3}{4}d.$
Ans. 230*l.* 0*s.* $2\frac{1}{4}d.$

5. Add together 471 cub. yds. 17 ft. 1549 in., 15 yds. 25 ft. 1718 in., 2 yds. 16 ft. 34 in., 315 yds. 3 ft. 2 in., 815 yds. 12 ft. 1671 in., and 6 yds. 2 ft. 8 in.
Ans. 1626 cub. yds. 23 ft. 1526 in.

6. From 678 miles 3 fur. 37 pol. 3 yds. take 189 miles 7 fur. 35 poles 4 yds. Ans. 488 mi. 4 fur. 1 po. $4\frac{1}{2}$ yds.

7. Multiply 649*l.* 18*s.* $4\frac{1}{2}d.$ by 87. Ans. 56542*l.* 18*s.* $7\frac{1}{2}d.$

8. $77\frac{3}{4}d$ yds. at 5*s.* 10*d.* per yard. Ans. 22*l.* 13*s.* $6\frac{1}{2}d.$

9. Divide 3059*l.* 17*s.* $6\frac{1}{2}d.$ by 132. Ans. 23*l.* 3*s.* $7\frac{1}{4}d.$

10. Divide 100*l.* into 8000 equal parts. Ans. 3*d.* each.

EXAMINATION PAPER. No. 3.

REDUCTION, PROPORTION, PRACTICE.

1. How many half crowns are equivalent to 253*l.* 9*s.* 10*d.*?

2. Find the number of cubic yards in 141721 cubic inches.

3. How often must a rod, 2 ft. 10 in. long, be applied to measure 10 miles 139½ yards?

4. Bought 27 yards for 11*l.*; how much will 44*l.* buy?

5. If 3 cwt. 2 qr. 11 lb. of tobacco cost 25*l.* 17*s.* 6¼*d.*, how much will be paid for 2 cwt. 1 qr.?

6. A vessel has provisions for 15 days, at the rate of 20 lb. per diem; what will be the daily rate if the voyage be calculated at 20 days, the provisions remaining the same?

7. Last year 30 men reaped a farm in 27 days; how many must be engaged to reap it this year in 10 days?

8. If the carriage of 14 cwt. 0 qr. 23 lb. for 65 miles comes to a certain sum, what weight may I have carried 37 miles for the same sum?

9. Find the cost of 2772 cwt., at 2*l.* 7*s.* 5¼*d.* per. cwt.

10. What is the value of 1 gallon of brandy, if 27¾ gallons cost 30*l.* 16*s.* 11*d.*?

11. If the circumference of the driving wheel of a locomotive be 16½ ft., how many revolutions will it make between Bristol and Exeter, the distance being 75½ miles?

EXAMINATION PAPER. No. 4.

FRACTIONS, DECIMALS, DUODECIMALS.

1. When can a vulgar fraction be exactly represented by a decimal? Can $\frac{12}{15}$?

2. Express ·0009375 by a vulgar fraction in its lowest terms.

Find the values of

3. 1·0000123 + 31·1 + 117·154 + 2343·008 + ·0002.

4. 3·35 — ·00098.

5. ·003 × ·01 × 500000.

6. 1·1 × ·011 × 1·01 × ·0101.

7. 4 ÷ ·00255.

8. $\cdot 2\dot{3} \times \cdot\dot{3}6$.

9. If 1 lb. of sugar cost ·0703125 of 16*s.*, what is the cost of ·0625 cwt.?

10. Find, by duodecimals, the expense of paving a yard which is 23 ft. 6 in. by 13 ft. 9 in., at 4*s.* 6*d.* the square yard.

EXAMINATION PAPER. No. 5.

INVOLUTION, EVOLUTION.

1. State two Nos. between 1 and 100, such that the 1st has exact square and cube roots, and the 2nd exact square and 4th roots.

Raise

2. $3\frac{1}{3}$ to the 4th power.

3. $\frac{\frac{1}{10}}{4\frac{2}{5}}$ to the 3rd power.

4. ·01 to the 6th power.

5. If a right-angled triangle has 10 feet in the hypothenuse, what are the other sides?

6. One end of a ladder, which is 15 feet long, just reaches the top of a wall, while the other end of it is 12 feet from the foot of the wall. Required the height of the wall.

Extract

7. The square root of 70756.

8. The cube root of 18821096.

9. The square root of ·00071289.

10. The square root of 266 to 7 places of decimals.

11. The cube root of 267 to 3 places of decimals.

EXAMINATION PAPER. No. 6.

COMMERCIAL RULES.

1. Required the simple interest on 847*l.* 16*s.* 8*d.* for 2 years, at 3 per cent.

2. Required the simple interest on 584*l.* 11*s.* 3½*d.* from Feb. 21 to Aug. 17 (1849), at 3½ per cent.

3. What is the brokerage on 439*l.* 12*s.* 6*d.*, at 3*s.* 4*d.* per cent.?

4. A bill dated Jan. 1, at 3 months date, for 739*l.* 16*s.* 11*d.* was discounted on Feb. 14, at 4 per cent.: what was the discount?

5. A person owes 100*l.* payable in 2 months, and 750*l.* payable in 7 months: what is the just time for the payment of the two debts *at once?*

6. A merchant buys 8½ tons of lead at 17*l.* 14*s.* 3*d.* per ton, and by selling gains 9*l.* on the whole: what was the selling price per cwt.?

7. A, B, C, D, purchase a ship: A pays $\frac{1}{3}$ of its value; B pays $\frac{5}{18}$; C pays $\frac{1}{6}$; D pays $\frac{2}{9}$. The net profit of a voyage is 364*l.* 17*s.* 6*d.*: how much of this sum ought each to receive?

8. Required the amount of 320*l.* 10*s.* for 4 years, at 5 per cent. compound interest.

9. Remitted from London to Amsterdam a bill for 7600*l.* 18*s.* sterling: what is the equivalent sum in Dutch money, exchange being 12 flor. 5½ stiv. per 1*l.* sterling?

10. How many francs must be paid at Paris to secure 160*l.* in London, exchange being 25 f. 70 c. per 1*l.* sterling?

11. A gentleman sold a house for 900 guineas, and by the bargain lost 5½ per cent.: how much was it sold for under its value, if it should have produced him 7 per cent. profit?

GENERAL EXAMINATION PAPER. No. 7.

1. *Partnership.* A, B, and C engage in trade; A puts in 128*l.*, B 176*l.*, and C 192*l.*: their profits amount to 279*l.*: what were their shares of it?

2. *Profit and Loss.* If tea be bought at 5*s.* 6*d.* per lb., and sold at 6*s.* 8*d.*, what is the gain per cent.?

3. *Discount.* What is the present value of 151*l.* 17*s.* 6*d.*, due at the end of 4 years, at $5\frac{3}{8}$ per cent.?

4. *Duodecimals.* Multiply 9 ft. 8 in. by 7 ft. 6 in.

5. *Square Root.* Extract the square roots of 33·64 and ·00038025.

6. *Cube Root.* Find the cube root of 185193.

7. *Decimals.* Divide 15·625 by 2·5, and ·03 by ·001.

8. ... Reduce 8*l.* 17*s.* $6\frac{3}{4}$*d.* to the decimal of 1*l.*

9. *Vulgar Fractions.* Reduce $111\frac{100}{111}$ to an improper fraction.

10. ... Reduce $\frac{22176}{23328}$ to its lowest terms.

11. ... Reduce $\frac{5}{8}$, $\frac{11}{12}$, $\frac{7}{18}$ to equivalent fractions having a common denominator.

12. ... Multiply $2\frac{1}{2}$ of $3\frac{3}{5}$ by $4\frac{3}{4}$ of $1\frac{1}{7}$; and divide $\frac{2}{5}$ of 5 by $\frac{3}{5}$ of $1\frac{1}{5}$.

13. ... Divide 30*l.* 14*s.* $6\frac{1}{2}$*d.* by $\frac{4}{23}$.

14. ... Reduce half-a-crown to the fraction of a half-a-guinea.

15. *Commission.* Find the commission on 768*l.* 2*s.* 6*d.*, at 3*s.* 4*d.* per cent.

16. *Interest.* Find the interest on 325*l.* 10*s.* for 4 years, at $5\frac{1}{2}$ per cent.

17. *Practice.* Find the rent of 8A. 3R. 10P. at 1*l.* 17*s.* 8*d.* per acre.

18. *Proportion.* If $1\frac{3}{8}$ cwt. sugar cost 7 guineas, what must be given for $17\frac{3}{4}$ lb.?

19. ... What is the income corresponding to an income tax of 13*l.* 2*s.* 6*d.*, at the rate of 7*d.* in the pound?

20. *Reduction.* Reduce 27 w. 5 d. 15 hrs. to seconds.

21. *Compound Division.* Divide 3587 yds. 9 in. by 27.

22. *First Rules.* Divide 6342576 by 24.

23. From 683125 take 492816, and prove your work

Miscellaneous.

24. Light travels at the rate of 192500 miles per second: if a ray from the sun takes 8 min. 13 sec. in reaching us, what is his distance from the earth?

25. Two boats start in a race, and one of them gains 5 feet upon the other in every 55 yards; how much will it have gained at the end of half-a-mile?

26. Find the least common multiple of 2, 4, 8, 16, 10, 48.

27. Compare the values of $\frac{1}{21}$ of half-a-crown, $\frac{1}{24}$ of 3*s.* 4*d.*, and $\frac{1}{28}$ of 4*s.* 2½*d.*

28. The dimensions of a room are 29½ ft. by 11¼ ft.; what will be the expense of covering it with carpet which is $\frac{5}{8}$ yd. wide, and costs 3*s.* 9*d.* per yard?

29. A person fails to the amount of 9000*l.*; his effects are worth only 3515*l.* 12*s.* 6*d.*; what will be the dividend in the pound? and what the loss upon a debt of 750*l.*?

30. What would be the difference of income, made by the transfer of 5000*l.* stock from the 3 per cents. at 72, to the 4 per cents. at 90?

31. Find the amount of 50*l.* for 3 years, at 5 per cent compound interest.

EXAMINATION PAPER. No. 8.

FRACTIONS AND DECIMALS.

1. Define a fraction. Define the different kinds of fractions. Prove that the value of a fraction is not altered by multiplying both its numerator and denominator by the same number.

2. Reduce $\frac{825}{960}$ to its lowest terms in two different ways.

3. Reduce $\frac{5}{2}$, $\frac{3}{4}$ of 5½, $4\frac{7}{11}$, and 5 to the least common denominator, and find their sum.

4. If $\frac{3}{8}$ of a merchant vessel are worth 1840*l.*, what must $\frac{5}{16}$ be sold for to gain 20*l.* by the sale?

5. What is a decimal fraction? What are the advantages in the use of decimal fractions? What effect has a cipher placed on the right hand side of a decimal fraction? What effect has it on the left hand side?

6. Divide ·00683468 by 2·0102, and reduce 6 cwt. 2 qrs. 7 lb. to the decimal of a ton.

EXAMINATION PAPER. No. 9.

SQUARE ROOT, CUBE ROOT, DUODECIMALS.

1. Find the square root of $51\frac{3}{4}$, and the cube root of 48228544.

2. The two slants of a gable are each 15 feet, and its perpendicular height is 9 feet: what is its breadth?

3. What will be the diameter of a circular plate of $\frac{1}{20}$th of an inch in thickness, which is to be beaten out from a sphere of metal of one foot diameter?

4. What must the length of a ladder be to reach across a ditch, 45 yards wide, to the top of a castle 30 yards high?

Ans. 54·08 yds.

5. Two ships sail from the same port: what is the distance between them, when one has sailed due north 80 leagues, and the other due west 60 leagues? Ans. 100 leagues.

6. A ladder of 100 feet in length was placed against a building, 100 feet high, in such a manner that the top of it reached the top of the building within 6 inches: what was the distance of the foot of the ladder from the base of the building? Ans. 9·9874921.

7. The area of a triangle is 240 feet: what is the length of each side of a square equal in area to the triangle?

Ans. 15·491933

8. Two trees on a horizontal plane are 120 feet distant from each other, one tree is 100 feet, and the other 80 feet high: how far from the base of each tree must a person stand that his distance from the top of each tree may be the same as the distance of the tops of the trees from each other?

9. The solid content of a sphere equals the cube of its diameter × ·5236: what is the solidity of a sphere of 20 inches diameter?

10. A cubic inch of glass is blown into the form of a globe that will hold 1 pint of wine: what is the thickness of the glass?

11. If the extreme end of the minute hand of a clock moves over a distance of five inches in 3 minutes 45 seconds, what is the length of the index?

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